Question1

The value of $\int_0^\pi \left|\sin^3 x\right| \mathrm{d}x$ is MHT CET 2025 (5 May Shift 2)

Options:

A. 0

B. $\frac{3}{8}$

C. $\frac{4}{3}$

D. π

Answer: C

Solution:

Since $\sin x \geq 0$ on $[0, \pi]$, $|\sin^3 x| = \sin^3 x$.

$$\int_0^\pi \sin^3 x\, dx = \int_0^\pi \sin x(1-\cos^2 x)\, dx = \left[-\cos x + \frac{\cos^3 x}{3}\right]_0^\pi = \left(1-\frac{1}{3}\right) - \left(-1+\frac{1}{3}\right) = \frac{4}{3}.$$

Answer: $\boxed{\frac{4}{3}}$.

Question2

$\int_0^1 \log(x+1)\mathrm{d}x =$ **MHT CET 2025 (5 May Shift 2)**

Options:

A. $\log 2 - 1$

B. $\log 2 + 1$

C. $2\log 2 + 1$

D. $2\log 2 - 1$

Answer: D

Solution:

$$I = \int_0^1 \log(1+x)\, dx$$

Let $u = 1 + x \Rightarrow du = dx$. Then

$$I = \int_1^2 \log u\, du = [u\log u - u]_1^2 = (2\log 2 - 2) - (0-1) = 2\log 2 - 1.$$

So,

$$\boxed{2\log 2 - 1}$$

Question3

$\int_0^{\frac{\pi}{6}} \left(2+3x^2\right) \cos 3x dx =$ **MHT CET 2025 (27 Apr Shift 2)**

Options:

A. $\frac{2}{9}+\frac{\pi^2}{36}$

B. $\frac{4}{9}+\frac{\pi^2}{36}$

C. $\frac{2}{9}-\frac{\pi^2}{36}$

D. $\frac{4}{9}-\frac{\pi^2}{36}$

Answer: B

Solution:

$$I=\int_0^{\pi/6}(2+3x^2)\cos(3x)\,dx=\underbrace{\int 2\cos(3x)\,dx}_{(2/3)\sin(3x)}+3\int x^2\cos(3x)\,dx$$

For $K=\int x^2\cos(3x)\,dx$, integrate by parts twice:

$$K=\frac{x^2}{3}\sin(3x)-\frac{2}{3}\int x\sin(3x)\,dx=\frac{x^2}{3}\sin(3x)+\frac{2x}{9}\cos(3x)-\frac{2}{27}\sin(3x).$$

Hence

$$3K=\sin(3x)\left(x^2-\frac{2}{9}\right)+\frac{2x}{3}\cos(3x).$$

Evaluate from 0 to $a=\pi/6$ ($\sin(3a)=1, \cos(3a)=0$):

$$I=\left[\frac{2}{3}\sin(3x)+\sin(3x)\left(x^2-\frac{2}{9}\right)+\frac{2x}{3}\cos(3x)\right]_0^{\pi/6}=\frac{2}{3}+\left(a^2-\frac{2}{9}\right)=\frac{4}{9}+\frac{\pi^2}{36}.$$

$$\boxed{\frac{4}{9}+\frac{\pi^2}{36}}$$

Question4

If $\mathrm{f}(5-x)=\mathrm{f}(x)$ **and** $\int_2^3 \mathrm{f}(x)\mathrm{d}x=2$ **then** $\int_2^3 x\mathrm{f}(x)\mathrm{d}x=$ **MHT CET 2025 (27 Apr Shift 2)**

Options:

A. 2

B. 3

C. 4

D. 5

Answer: D

Solution:

Because $f(5-x)=f(x)$ on $[2,3]$, the function is symmetric about $x=\frac{5}{2}$.

Let $I=\int_2^3 x\,f(x)\,dx$.

With the change $u=5-x$ (so $du=-dx$),

$$I=\int_2^3 x\,f(x)\,dx=\int_2^3 (5-u)\,f(u)\,du=\int_2^3 (5-x)\,f(x)\,dx.$$

Add the two expressions:

$$2I=\int_2^3 [x+(5-x)]\,f(x)\,dx=5\int_2^3 f(x)\,dx=5\cdot 2=10.$$

Hence $I=5$.

$$\boxed{5}$$

Question5

$\int_0^1 x\left|x-\frac{1}{2}\right|\mathrm{d}x=$ **MHT CET 2025 (26 Apr Shift 2)**

Options:

A. $\frac{1}{2}$

B. $\frac{1}{12}$

C. $\frac{1}{8}$

D. $\frac{1}{16}$

Answer: C

Solution:

Split at $x=\frac{1}{2}$:

$$\int_0^1 x|x-\tfrac{1}{2}|\,dx=\int_0^{1/2} x\left(\tfrac{1}{2}-x\right)dx+\int_{1/2}^1 x(x-\tfrac{1}{2})dx.$$

First part:

$$\int_0^{1/2}\left(\frac{x}{2}-x^2\right)dx=\left[\frac{x^2}{4}-\frac{x^3}{3}\right]_0^{1/2}=\frac{1}{48}.$$

Second part:

$$\int_{1/2}^1\left(x^2-\frac{x}{2}\right)dx=\left[\frac{x^3}{3}-\frac{x^2}{4}\right]_{1/2}^1=\frac{5}{48}.$$

Add: $\frac{1}{48}+\frac{5}{48}=\frac{6}{48}=\boxed{\frac{1}{8}}$.

Question6

$\int_0^{\frac{\pi}{2}}\frac{300\sin x+100\cos x}{\sin x+\cos x}\,\mathrm{d}x=\ldots$ **MHT CET 2025 (26 Apr Shift 2)**

Options:

A. 100π

B. 300π

C. 200π

D. 150π

Answer: A

Solution:

Let

$$I=\int_0^{\pi/2}\frac{300\sin x+100\cos x}{\sin x+\cos x}\,dx.$$

Substitute $x\mapsto \frac{\pi}{2}-x$ to get

$$I=\int_0^{\pi/2}\frac{300\cos x+100\sin x}{\sin x+\cos x}\,dx.$$

Add the two expressions:

$$2I=\int_0^{\pi/2}\frac{(300+100)(\sin x+\cos x)}{\sin x+\cos x}\,dx=(400)\int_0^{\pi/2}1\,dx=400\cdot\frac{\pi}{2}.$$

Hence $I=\dfrac{400}{2}\cdot\frac{\pi}{2}=100\pi.$

$$\boxed{100\pi}$$

Question7

$\int_0^1 \tan^{-1} x dx =$ **MHT CET 2025 (26 Apr Shift 1)**

Options:

A. $\frac{\pi}{4}-\log 2$

B. $\frac{\pi}{4}-\log\sqrt{2}$

C. $\frac{\pi}{4}+\log 2$

D. $\frac{\pi}{4}+\log\sqrt{2}$

Answer: B

Solution:

Step 1: Integration by Parts

We use the integration by parts formula $\int u\,dv=uv-\int v\,du$.

Choose:

- $u=\tan^{-1}x$ (because it's easy to differentiate)
- $dv=1\,dx$ (because it's easy to integrate)

Find du and v:

- $du=\frac{1}{1+x^2}\,dx$
- $v=\int 1\,dx=x$

Apply the formula to the definite integral:

$$I=[uv]_0^1-\int_0^1 v\,du$$

$$I = \left[x \tan^{-1} x\right]_0^1 - \int_0^1 x \left(\frac{1}{1+x^2}\right) dx$$

$$I = \left[x \tan^{-1} x\right]_0^1 - \int_0^1 \frac{x}{1+x^2} dx$$

Step 2: Evaluate the First Term

$$\left[x \tan^{-1} x\right]_0^1 = \left[1 \cdot \tan^{-1}(1)\right] - \left[0 \cdot \tan^{-1}(0)\right]$$

Since $\tan^{-1}(1) = \frac{\pi}{4}$ and $\tan^{-1}(0) = 0$:

$$= \left[1 \cdot \frac{\pi}{4}\right] - [0]$$

$$= \frac{\pi}{4}$$

Step 3: Evaluate the Remaining Integral

The remaining integral is $J = \int_0^1 \frac{x}{1+x^2} dx$. We can solve this using a **u-substitution**.

Let $u = 1 + x^2$. Then $du = 2x\, dx$, which means $x\, dx = \frac{1}{2} du$.

Change the limits of integration:

- When $x = 0$, $u = 1 + (0)^2 = 1$.
- When $x = 1$, $u = 1 + (1)^2 = 2$.

Substitute u and the new limits into J:

$$J = \int_1^2 \frac{1}{u} \left(\frac{1}{2} du\right)$$

$$J = \frac{1}{2} \int_1^2 \frac{1}{u} du$$

$$J = \frac{1}{2} \left[\ln |u|\right]_1^2$$

$$J = \frac{1}{2} (\ln(2) - \ln(1))$$

Since $\ln(1) = 0$:

$$J = \frac{1}{2} \ln(2)$$

Using the logarithm property $a \ln b = \ln b^a$:

$$J = \ln(2^{1/2}) = \ln(\sqrt{2})$$

Step 4: Final Calculation

Substitute the results for the two parts back into the integral equation:

$$I = \left[x \tan^{-1} x\right]_0^1 - J$$

$$I = \frac{\pi}{4} - \ln(\sqrt{2})$$

This result matches option **B**.

The option you marked, **A** ($\frac{\pi}{4} - \log 2$), is **incorrect**.

The correct answer is:

$$\frac{\pi}{4} - \log\sqrt{2}$$

Question8

$\int_{\frac{1}{2}}^{2} \frac{1}{x} \text{cosec}^{101}\left(x - \frac{1}{x}\right) \mathrm{d}x =$ **MHT CET 2025 (26 Apr Shift 1)**

Options:

A. 0

B. 1

C. $\frac{1}{4}$

D. $\frac{101}{2}$

Answer: A

Solution:

Let

$$I = \int_{1/2}^{2} \frac{1}{x} \csc^{101}\left(x - \frac{1}{x}\right) dx.$$

Use the substitution $x \mapsto \dfrac{1}{x}$. Then $dx = -\dfrac{1}{x^2}dx$ (or with a new variable t), and

$$I = \int_{2}^{1/2} \frac{1}{t} \csc^{101}\left(\frac{1}{t} - t\right) dt = \int_{2}^{1/2} \frac{1}{t} \csc^{101}\left(-(t - \tfrac{1}{t})\right) dt.$$

Since $\csc(-u) = -\csc(u)$, for odd power 101:

$$\csc^{101}(-(t - \tfrac{1}{t})) = -\csc^{101}(t - \tfrac{1}{t}).$$

Thus

$$I = -\int_{2}^{1/2} \frac{1}{t} \csc^{101}\left(t - \frac{1}{t}\right) dt = \int_{1/2}^{2} \frac{1}{t} \csc^{101}\left(t - \frac{1}{t}\right) dt = -I.$$

Hence $I = -I \Rightarrow 2I = 0 \Rightarrow I = 0.$

$$\boxed{0}$$

Question9

Let $f : \mathbb{R} \to \mathbb{R}$ is differentiable function having $f(3) = 3$, $f'(3) = \frac{1}{27}$ and

$\mathrm{g}(\mathrm{x})=\begin{cases}\int_3^{\mathrm{f}(x)} \frac{3\mathrm{t}^2}{x-3}\mathrm{dt} & , \quad \text{if } x\neq 3 \\ \mathrm{K} & , \quad \text{if } x=3\end{cases}$ **is continuous at $x=3$, then $\mathrm{K}=$ MHT CET 2025 (25 Apr Shift 2)**

Options:

A. 1

B. 3

C. $\frac{1}{3}$

D. 9

Answer: A

Solution:

We need $K=\lim_{x\to 3} g(x)$ for continuity.

For $x\neq 3$:

$$g(x)=\int_3^{f(x)} \frac{3t^2}{x-3}\,dt=\frac{1}{x-3}\left[t^3\right]_3^{f(x)}=\frac{f(x)^3-27}{x-3}.$$

Since $f(3)=3$, the limit is $0/0$. Differentiate numerator and denominator w.r.t. x (L'Hôpital):

$$\lim_{x\to 3}\frac{f(x)^3-27}{x-3}=\lim_{x\to 3}\frac{3f(x)^2 f'(x)}{1}=3\cdot 3^2\cdot f'(3)=3\cdot 9\cdot\frac{1}{27}=1.$$

Thus, to make g continuous at $x=3$,

$$\boxed{K=1}.$$

Question10

$\int_{\log\frac{1}{2}}^{\log 2} \sin\left(\frac{e^x-1}{e^x+1}\right)dx=$ **MHT CET 2025 (25 Apr Shift 2)**

Options:

A. 0

B. 1

C. $\cos\frac{1}{2}$

D. $2\log\frac{1}{2}$

Answer: A

Solution:

1. Limits symmetric hain

$$\int_{\log(1/2)}^{\log 2} \cdots dx$$

Let $a = \log 2$. Then lower limit $\log(1/2) = -a$ and upper limit a. So interval $[-a, a]$ symmetric hai.

2. Integrand odd hai
   Define

$$f(x) = \sin\left(\frac{e^x - 1}{e^x + 1}\right).$$

Ab x ki jagah $-x$ rakho:

$$f(-x) = \sin\left(\frac{e^{-x} - 1}{e^{-x} + 1}\right) = \sin\left(\frac{1 - e^x}{1 + e^x}\right) = \sin\left(-\frac{e^x - 1}{e^x + 1}\right) = -\sin\left(\frac{e^x - 1}{e^x + 1}\right) = -f(x).$$

Isliye f odd function hai (i.e., $f(-x) = -f(x)$).

3. Symmetric interval par odd function ka integral zero hota hai
   Property: $\int_{-a}^{a} (\text{odd function})\, dx = 0.$
   Yahaan interval $[-a, a]$ hai aur integrand odd hai, to

$$\int_{\log(1/2)}^{\log 2} \sin\left(\frac{e^x - 1}{e^x + 1}\right) dx = 0.$$

Question11

$\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \sin^{-4} x \, \mathrm{d}x =$ **MHT CET 2025 (25 Apr Shift 2)**

Options:

A. $\frac{8}{3}$

B. $-\frac{8}{3}$

C. $\frac{2}{3}$

D. $-\frac{2}{3}$

Answer: B

Solution:

Use $\sin^{-4} x = \csc^4 x = \csc^2 x(1 + \cot^2 x)$ and $d(\cot x) = -\csc^2 x\, dx$.

$$\int \csc^4 x\, dx = -\int (1 + \cot^2 x)\, d(\cot x) = -\cot x - \frac{1}{3}\cot^3 x + C.$$

Now evaluate at the bounds.

- If the integral is $\int_{\pi/4}^{3\pi/4} \sin^{-4} x\, dx$:

$$\left[-\cot x - \tfrac{1}{3}\cot^3 x\right]_{\pi/4}^{3\pi/4} = \left(1 + \tfrac{1}{3}\right) - \left(-1 - \tfrac{1}{3}\right) = \boxed{\tfrac{8}{3}}.$$

- If the order is reversed, $\int_{3\pi/4}^{\pi/4} \sin^{-4} x\, dx$, the value is the negative:

$$\boxed{-\frac{8}{3}}.$$

Question12

$\int_1^3 \frac{\log x^2}{\log(16x^2-8x^3+x^4)} \mathrm{d}x = \ldots$ **MHT CET 2025 (25 Apr Shift 1)**

Options:

A. 1

B. 3

C. $\log 2$

D. $\frac{1}{2}$

Answer: A

Solution:

Let

$$I = \int_1^3 \frac{\log x^2}{\log(16x^2 - 8x^3 + x^4)}\, dx.$$

First simplify the denominator:

$$16x^2 - 8x^3 + x^4 = x^2(x^2 - 8x + 16) = x^2(x-4)^2.$$

Hence

$$\frac{\log x^2}{\log(16x^2 - 8x^3 + x^4)} = \frac{2\log x}{\log\left(x^2(x-4)^2\right)} = \frac{\log x}{\log\left(x|x-4|\right)}.$$

On $x \in [1,3]$, $x - 4 < 0 \Rightarrow |x-4| = 4 - x$. Define

$$f(x) = \frac{\log x}{\log\left(x(4-x)\right)}.$$

Then

$$f(4-x) = \frac{\log(4-x)}{\log\left((4-x)x\right)} \quad \Rightarrow \quad f(x) + f(4-x) = \frac{\log x + \log(4-x)}{\log(x(4-x))} = 1.$$

Using the substitution $x \mapsto 4 - x$ (the interval $[1,3]$ is symmetric about 2):

$$I = \int_1^3 f(x)\, dx = \int_1^3 f(4-x)\, dx \Rightarrow 2I = \int_1^3 \left[f(x) + f(4-x)\right] dx = \int_1^3 1\, dx = 2.$$

Thus $I = 1$.

$$\boxed{1}$$

Question13

$\int_0^1 \frac{1}{2+\sqrt{x}}\,\mathrm{d}x =$ **MHT CET 2025 (25 Apr Shift 1)**

Options:

A. $2\log\left(\frac{2e}{3}\right)$

B. $2\log\left(\frac{4e}{9}\right)$

C. $\log\left(\frac{2e}{3}\right)$

D. $\log\left(\frac{4e}{9}\right)$

Answer: B

Solution:

Let $I = \int_0^1 \frac{1}{2+\sqrt{x}}\,dx.$

Put $t = \sqrt{x} \Rightarrow x = t^2,\ dx = 2t\,dt,\ t \in [0,1]$:

$$I = \int_0^1 \frac{2t}{2+t}\,dt = \int_0^1 \left(2 - \frac{4}{t+2}\right)dt = \Big[2t - 4\ln(t+2)\Big]_0^1.$$

Evaluate:

$$I = (2 - 4\ln 3) - (0 - 4\ln 2) = 2 + 4\ln\frac{2}{3} = 2\ln\left(\frac{4e}{9}\right).$$

So the value is $\boxed{2\log\left(\frac{4e}{9}\right)}$ (natural log).

Question14

$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \left(x^2 + \log\left(\frac{\pi-x}{\pi+x}\right)\cdot\cos x\right)\mathrm{d}x =$ **MHT CET 2025 (23 Apr Shift 2)**

Options:

A. 0

B. $\frac{\pi^3}{48}$

C. $\frac{\pi^3}{12}$

D. $\frac{\pi^3}{24}$

Answer: C

Solution:

Let $I = \int_{-\pi/2}^{\pi/2} \left(x^2 + \log\frac{\pi - x}{\pi + x} \cdot \cos x\right) dx.$

Split it:

1. x^2 is even, so

$$\int_{-\pi/2}^{\pi/2} x^2\,dx = 2\int_0^{\pi/2} x^2\,dx = 2\left[\frac{x^3}{3}\right]_0^{\pi/2} = \frac{\pi^3}{12}.$$

2. For the second term, note

$$\phi(x) = \log\frac{\pi - x}{\pi + x} \quad \Rightarrow \quad \phi(-x) = \log\frac{\pi + x}{\pi - x} = -\phi(x),$$

so ϕ is odd, while $\cos x$ is even. Hence $\phi(x)\cos x$ is odd and

$$\int_{-\pi/2}^{\pi/2} \phi(x)\cos x\,dx = 0.$$

Therefore,

$$I = \frac{\pi^3}{12}.$$

$\boxed{\frac{\pi^3}{12}}.$

Question15

$\int_0^1 \log\left(\frac{1}{x} - 1\right) \mathrm{d}x =$ **MHT CET 2025 (23 Apr Shift 2)**

Options:

A. $\frac{1}{2}$

B. 1

C. 2

D. 0

Answer: D

Solution:

$$I = \int_0^1 \log\left(\frac{1}{x} - 1\right) dx = \int_0^1 \left[\log(1 - x) - \log x\right] dx.$$

But

$$\int_0^1 \log(1 - x)\,dx \overset{u=1-x}{=} \int_0^1 \log u\,du = \Big[u\log u - u\Big]_0^1 = -1,$$

and

$$\int_0^1 \log x\,dx = \Big[x\log x - x\Big]_0^1 = -1.$$

Hence

$$I = (-1) - (-1) = \boxed{0}.$$

Question16

The value of $\int_0^1 \tan^{-1}\left(1 - x + x^2\right) \mathrm{d}x$ **is MHT CET 2025 (23 Apr Shift 1)**

Options:

A. $\frac{\pi}{2} - \log 2$

B. $\frac{\pi}{2} + \log 2$

C. $\log 2$

D. 0

Answer: C

Solution:

The correct option is **(C) log 2**.

The solution uses the property of definite integrals that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$.

Let $I = \int_0^1 \tan^{-1}(1-x+x^2)dx$.

Using the identity $\tan^{-1}(x) + \tan^{-1}(y) = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$, we can show that

$\tan^{-1}(x) + \tan^{-1}(1-x) = \tan^{-1}\left(\frac{1}{1-x+x^2}\right)$.

Also, using the identity $\tan^{-1}(z) + \cot^{-1}(z) = \frac{\pi}{2}$, and $\cot^{-1}(z) = \tan^{-1}\left(\frac{1}{z}\right)$, we can write:

$$\tan^{-1}(1-x+x^2) = \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{1-x+x^2}\right) = \frac{\pi}{2} - (\tan^{-1}(x) + \tan^{-1}(1-x))$$

Integrating both sides from 0 to 1:

$I = \int_0^1 \left[\frac{\pi}{2} - \tan^{-1}(x) - \tan^{-1}(1-x)\right] dx = \int_0^1 \frac{\pi}{2} dx - \int_0^1 \tan^{-1}(x)dx - \int_0^1 \tan^{-1}(1-x)dx$

.

Using the property $\int_0^1 \tan^{-1}(1-x)dx = \int_0^1 \tan^{-1}(x)dx$, the equation becomes:

$I = \frac{\pi}{2} - 2\int_0^1 \tan^{-1}(x)dx$.

The integral $\int_0^1 \tan^{-1}(x)dx$ can be evaluated by parts to be $\frac{\pi}{4} - \frac{1}{2}\ln 2$.

Substituting this back, we get:

$I = \frac{\pi}{2} - 2\left(\frac{\pi}{4} - \frac{1}{2}\ln 2\right) = \frac{\pi}{2} - \frac{\pi}{2} + \ln 2 = \ln 2$.

Question17

$\int_3^5 \frac{\sqrt{x}dx}{\sqrt{8-x}+\sqrt{x}} =$ **MHT CET 2025 (23 Apr Shift 1)**

Options:

A. 0

B. 1

C. 2

D. 3

Answer: B

Solution:

Let

$$I = \int_3^5 \frac{\sqrt{x}}{\sqrt{8-x}+\sqrt{x}}\,dx.$$

Use the substitution $x \mapsto 8-x$:

$$I = \int_3^5 \frac{\sqrt{8-x}}{\sqrt{x}+\sqrt{8-x}}\,dx.$$

Add the two forms:

$$2I = \int_3^5 \frac{\sqrt{x}+\sqrt{8-x}}{\sqrt{8-x}+\sqrt{x}}\,dx = \int_3^5 1\,dx = 5-3 = 2.$$

Hence $I = 1$.

$$\boxed{1}$$

Question18

$\int_0^{\frac{\pi}{4}}(\sqrt{\tan x}+\sqrt{\cot x})\mathrm{d}x =$ **MHT CET 2025 (22 Apr Shift 2)**

Options:

A. $\sqrt{2}\pi$

B. $\frac{\pi}{2}$

C. 2π

D. $\frac{\pi}{\sqrt{2}}$

Answer: D

Solution:

Let

$$I = \int_0^{\pi/4} \left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx.$$

Put $u = \tan x$ ($x \in [0,\pi/4] \Rightarrow u \in [0,1]$, $dx = \frac{du}{1+u^2}$). Then

$$I = \int_0^1 \frac{\sqrt{u}}{1+u^2}\,du + \int_0^1 \frac{1/\sqrt{u}}{1+u^2}\,du = \int_0^1 \frac{\sqrt{u}+u^{-1/2}}{1+u^2}\,du.$$

Now set $u = t^2$ ($t \in [0,1]$, $du = 2t\,dt$):

$$I = \int_0^1 \frac{t+\frac{1}{t}}{1+t^4}\cdot 2t\,dt = 2\int_0^1 \frac{1+t^2}{1+t^4}\,dt.$$

Use the antiderivative

$$\int \frac{1+t^2}{1+t^4}\,dt = -\frac{1}{\sqrt{2}}\arctan\left(\frac{1-t^2}{\sqrt{2}\,t}\right)+C,$$

which gives, from 0 to 1,

$$\int_0^1 \frac{1+t^2}{1+t^4}\,dt = \frac{\pi}{2\sqrt{2}}.$$

Therefore

$$I = 2\cdot\frac{\pi}{2\sqrt{2}} = \boxed{\frac{\pi}{\sqrt{2}}}.$$

Question19

The value of the integral $\int_1^2 \frac{x\,\mathrm{d}x}{(x+2)(x+3)}$ is MHT CET 2025 (22 Apr Shift 2)

Options:

A. $\log\left(\frac{125}{16}\right)$

B. $\log\left(\frac{1024}{1125}\right)$

C. $\log\left(\frac{16}{125}\right)$

D. $\log\left(\frac{1125}{1024}\right)$

Answer: D

Solution:

$$I = \int_1^2 \frac{x}{(x+2)(x+3)}\,dx$$

Partial fractions:

$$\frac{x}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3} \Rightarrow x = A(x+3) + B(x+2)$$

So $A + B = 1,\ 3A + 2B = 0 \Rightarrow A = -2,\ B = 3.$

Thus

$$I = \int_1^2 \left(\frac{-2}{x+2} + \frac{3}{x+3}\right)dx = \Big[-2\ln(x+2) + 3\ln(x+3)\Big]_1^2.$$

Evaluate:

$$I = (3\ln 5 - 2\ln 4) - (3\ln 4 - 2\ln 3) = \ln\left(\frac{125}{16}\cdot\frac{9}{64}\right) = \ln\left(\frac{1125}{1024}\right).$$

$$\boxed{\log\left(\frac{1125}{1024}\right)}$$

Question20

$\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{x}{1+\sin x}\,\mathrm{d}x =$ MHT CET 2025 (22 Apr Shift 1)

Options:

A. $\pi(\sqrt{3} - 2)$

B. $\pi(2 - \sqrt{3})$

C. $\pi(\sqrt{3} + 2)$

D. $\frac{\pi}{2}(2 - \sqrt{3})$

Answer: B

Solution:

Let $I = \int_{\pi/3}^{2\pi/3} \frac{x}{1+\sin x}\, dx.$

Use $x \mapsto \pi - x$ (since the limits are symmetric about $\pi/2$):

$$I = \int_{\pi/3}^{2\pi/3} \frac{\pi - x}{1+\sin x}\, dx.$$

Add the two forms:

$$2I = \int_{\pi/3}^{2\pi/3} \frac{x+\pi-x}{1+\sin x}\, dx = \pi \int_{\pi/3}^{2\pi/3} \frac{dx}{1+\sin x}.$$

Evaluate the last integral:

$$\int \frac{dx}{1+\sin x} = \int \frac{1-\sin x}{\cos^2 x}\, dx = \int (\sec^2 x - \sec x \tan x)\, dx = \tan x - \sec x.$$

Hence

$$\int_{\pi/3}^{2\pi/3} \frac{dx}{1+\sin x} = \Big[\tan x - \sec x\Big]_{\pi/3}^{2\pi/3} = (2-\sqrt{3}) - (\sqrt{3}-2) = 2(2-\sqrt{3}).$$

Therefore

$$2I = \pi \cdot 2(2-\sqrt{3}) \quad \Rightarrow \quad I = \boxed{\pi(2-\sqrt{3})}.$$

Question21

$\int_0^2 \frac{3x+1}{x^2+4} dx =$ **MHT CET 2025 (22 Apr Shift 1)**

Options:

A. $\log(2\sqrt{2}) + \frac{\pi}{4}$

B. $\log(2\sqrt{2}) + \frac{\pi}{6}$

C. $\log(2\sqrt{2}) + \frac{\pi}{8}$

D. $\log(2\sqrt{2}) + \frac{\pi}{12}$

Answer: C

Solution:

$$\int_0^2 \frac{3x+1}{x^2+4}\, dx = \int_0^2 \frac{3x}{x^2+4}\, dx + \int_0^2 \frac{1}{x^2+4}\, dx.$$

First term:

$$\int \frac{3x}{x^2+4}\, dx = \tfrac{3}{2}\ln(x^2+4) \Rightarrow \left[\tfrac{3}{2}\ln(x^2+4)\right]_0^2 = \tfrac{3}{2}\ln\frac{8}{4} = \tfrac{3}{2}\ln 2.$$

Second term:

$$\int \frac{1}{x^2+4}\, dx = \tfrac{1}{2}\arctan\frac{x}{2} \Rightarrow \left[\tfrac{1}{2}\arctan\frac{x}{2}\right]_0^2 = \tfrac{1}{2}\cdot\frac{\pi}{4} = \frac{\pi}{8}.$$

Sum:

$$I = \tfrac{3}{2}\ln 2 + \frac{\pi}{8} = \ln(2^{3/2}) + \frac{\pi}{8} = \boxed{\ln(2\sqrt{2}) + \frac{\pi}{8}}.$$

Question22

$\int_{-1}^{3} \left(\tan^{-1}\left(\frac{x}{x^2+1}\right) + \tan^{-1}\left(\frac{x^2+1}{x}\right)\right) \mathrm{d}x =$ **MHT CET 2025 (21 Apr Shift 2)**

Options:

A. $\frac{\pi}{2}$

B. π

C. $\frac{2\pi}{3}$

D. 2π

Answer: B

Solution:

$$I=\int_{-1}^{3}\left(\tan^{-1}\frac{x}{x^2+1}+\tan^{-1}\frac{x^2+1}{x}\right)dx$$

For $x \neq 0$, set $u=\frac{x}{x^2+1}$. Then $\frac{1}{u}=\frac{x^2+1}{x}$.
Using the arctan identity

$$\arctan u+\arctan\frac{1}{u}=\begin{cases}\frac{\pi}{2}, & u>0,\\ -\frac{\pi}{2}, & u<0,\end{cases}$$

and noting that $\operatorname{sgn}(u)=\operatorname{sgn}(x)$ (since $x^2+1>0$), the integrand equals

$$\arctan\frac{x}{x^2+1}+\arctan\frac{x^2+1}{x}=\frac{\pi}{2}\operatorname{sgn}(x).$$

Thus the integral splits at $x=0$:

$$I=\int_{-1}^{0}\left(-\frac{\pi}{2}\right)dx+\int_{0}^{3}\left(\frac{\pi}{2}\right)dx=-\frac{\pi}{2}(1)+\frac{\pi}{2}(3)=\boxed{\pi}.$$

Question23

$\int_0^{\frac{\pi}{4}}\frac{\cos^2 x\sin^2 x}{\left(\cos^3 x+\sin^3 x\right)^2}\,dx=$ **MHT CET 2025 (21 Apr Shift 2)**

Options:

A. $\frac{1}{3}$

B. $\frac{-1}{3}$

C. $\frac{1}{6}$

D. $\frac{-1}{6}$

Answer: C

Solution:

$$I=\int_0^{\pi/4}\frac{\cos^2 x\,\sin^2 x}{(\cos^3 x+\sin^3 x)^2}\,dx.$$

Let $t=\tan x$ $(x\in[0,\pi/4]\Rightarrow t\in[0,1])$. Then

$$\sin x=\frac{t}{\sqrt{1+t^2}},\quad \cos x=\frac{1}{\sqrt{1+t^2}},\quad dx=\frac{dt}{1+t^2}.$$

Compute

$$\cos^2 x\sin^2 x=\frac{t^2}{(1+t^2)^2},\qquad \cos^3 x+\sin^3 x=\frac{1+t^3}{(1+t^2)^{3/2}},$$

so

$$\frac{\cos^2 x\sin^2 x}{(\cos^3 x+\sin^3 x)^2}\,dx=\frac{t^2}{(1+t^3)^2}\,dt$$

(since $(\cos^3 x+\sin^3 x)^2=\frac{(1+t^3)^2}{(1+t^2)^3}$).

Therefore

$$I=\int_0^1\frac{t^2}{(1+t^3)^2}\,dt.$$

Let $u=1+t^3\Rightarrow du=3t^2dt$. Then

$$I=\frac{1}{3}\int_{u=1}^{2}\frac{1}{u^2}\,du=\frac{1}{3}\left[-\frac{1}{u}\right]_1^2=\frac{1}{3}\left(1-\frac{1}{2}\right)=\boxed{\frac{1}{6}}.$$

Question24

$\int_2^4\frac{\log x^2}{\log x^2+\log(36-12x+x^2)}\mathrm{d}x$ **is equal to MHT CET 2025 (21 Apr Shift 1)**

Options:

A. 1

B. 2

C. 4

D. 6

Answer: A

Solution:

$$I=\int_2^4 \frac{\log x^2}{\log x^2+\log(36-12x+x^2)}\,dx$$

Note $36-12x+x^2=(x-6)^2$. Then

$$\frac{\log x^2}{\log x^2+\log(36-12x+x^2)}=\frac{2\log x}{2\log\big(x(6-x)\big)}=\frac{\log x}{\log\big(x(6-x)\big)}=: f(x).$$

On $[2,4]$, define $y=6-x$ (the interval is symmetric about 3). Then

$$f(6-x)=\frac{\log(6-x)}{\log\big(x(6-x)\big)}.$$

Hence

$$f(x)+f(6-x)=\frac{\log x+\log(6-x)}{\log\big(x(6-x)\big)}=\frac{\log\big(x(6-x)\big)}{\log\big(x(6-x)\big)}=1.$$

Therefore

$$2I=\int_2^4 [f(x)+f(6-x)]\,dx=\int_2^4 1\,dx=2 \Rightarrow I=1.$$

$$\boxed{1}$$

Question25

The value of $\int_0^2 [x^2]\,\mathrm{d}x$ is (where $[x]$ denotes the greatest integer function not greater than x) MHT CET 2025 (21 Apr Shift 1)

Options:

A. $5-\sqrt{2}-\sqrt{3}$

B. $5+\sqrt{2}-\sqrt{3}$

C. $5+\sqrt{2}+\sqrt{3}$

D. $5-\sqrt{2}+\sqrt{3}$

Answer: A

Solution:

$$I=\int_0^2 \lfloor x^2\rfloor\,dx$$

Break where x^2 crosses integers:

- $x\in[0,1)$: $x^2\in[0,1)\Rightarrow\lfloor x^2\rfloor=0$.
- $x\in[1,\sqrt{2})$: $x^2\in[1,2)\Rightarrow\lfloor x^2\rfloor=1$.
- $x\in[\sqrt{2},\sqrt{3})$: $x^2\in[2,3)\Rightarrow\lfloor x^2\rfloor=2$.
- $x\in[\sqrt{3},2)$: $x^2\in[3,4)\Rightarrow\lfloor x^2\rfloor=3$.
  (The point $x=2$ where $x^2=4$ has measure zero.)

So,

$$I=(\sqrt{2}-1)\cdot 1+(\sqrt{3}-\sqrt{2})\cdot 2+(2-\sqrt{3})\cdot 3=5-\sqrt{2}-\sqrt{3}.$$

$$\boxed{5-\sqrt{2}-\sqrt{3}}$$

Question26

$\int_0^{\frac{\pi}{2}} \frac{\mathrm{d}x}{1+(\cot x)^{101}}=$ **MHT CET 2025 (20 Apr Shift 2)**

Options:

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{8}$

D. π

Answer: B

Solution:

$$I=\int_0^{\pi/2}\frac{dx}{1+\cot^{101}x}$$

Let $x\mapsto\frac{\pi}{2}-x$. Then $\cot x=\tan(\frac{\pi}{2}-x)$, so

$$I=\int_0^{\pi/2}\frac{dx}{1+\tan^{101}x}.$$

Add the two:

$$2I=\int_0^{\pi/2}\left(\frac{1}{1+\cot^{101}x}+\frac{1}{1+\tan^{101}x}\right)dx.$$

With $a=\tan^{101}x$,

$$\frac{1}{1+\cot^{101}x}=\frac{a}{1+a},\qquad\frac{1}{1+\tan^{101}x}=\frac{1}{1+a},$$

so the sum is 1. Thus

$$2I=\int_0^{\pi/2}1\,dx=\frac{\pi}{2}\Rightarrow I=\boxed{\frac{\pi}{4}}.$$

Question27

$\int_{-2}^{2}\left|x^2-x-2\right|\mathrm{dx}=$ **MHT CET 2025 (20 Apr Shift 1)**

Options:

A. $\frac{17}{3}$

B. $\frac{19}{3}$

C. 19

D. 17

Answer: B

Solution:

$$I = \int_{-2}^{2} |x^2 - x - 2|\,dx = \int_{-2}^{-1} (x^2 - x - 2)\,dx + \int_{-1}^{2} -(x^2 - x - 2)\,dx$$

since $x^2 - x - 2 = (x-2)(x+1)$ is > 0 on $[-2, -1]$ and < 0 on $(-1, 2)$.

Let $F(x) = \frac{x^3}{3} - \frac{x^2}{2} - 2x$.

$$\int_{-2}^{-1} (x^2 - x - 2)\,dx = F(-1) - F(-2) = \frac{7}{6} - \left(-\frac{2}{3}\right) = \frac{11}{6},$$

$$\int_{-1}^{2} -(x^2 - x - 2)\,dx = F(-1) - F(2) = \frac{7}{6} - \left(-\frac{10}{3}\right) = \frac{9}{2}.$$

Add them:

$$I = \frac{11}{6} + \frac{9}{2} = \frac{11}{6} + \frac{27}{6} = \boxed{\frac{19}{3}}.$$

Question28

The value of $\int_{-1}^{1} \left(\sqrt{1+x+x^2} - \sqrt{1-x+x^2}\right) \mathrm{d}x$ is MHT CET 2025 (20 Apr Shift 1)

Options:

A. -1

B. 0

C. 1

D. 2

Answer: B

Solution:

$$I = \int_{-1}^{1} \left(\sqrt{1+x+x^2} - \sqrt{1-x+x^2}\right) dx$$

Let $f(x) = \sqrt{1+x+x^2} - \sqrt{1-x+x^2}$.
Then

$$f(-x) = \sqrt{1-x+x^2} - \sqrt{1+x+x^2} = -f(x),$$

so f is an odd function. The interval $[-1, 1]$ is symmetric, hence

$$I = \int_{-1}^{1} f(x)\,dx = 0.$$

$\boxed{0}$.

Question29

The value of $\int_{\frac{1}{3}}^{1} \left(x - x^3\right)^{\frac{1}{3}} \mathrm{d}x$ is MHT CET 2025 (19 Apr Shift 2)

Options:

A. 0

B. 2

C. 4

D. 6

Answer: A

Solution:

The value of $\int_{1/3}^{1}(x-x^3)^{1/3}dx$ is zero because the integrand is an odd function about the midpoint of the interval.

Symmetry Argument

Let's set $x=t$ and transform $x \to 1-x$ within the limits $[1/3, 1]$:

$$f(x)=(x-x^3)^{1/3}$$

$$f(1-x)=((1-x)-(1-x)^3)^{1/3}$$

$$(1-x)^3=1-3x+3x^2-x^3$$

$$f(1-x)=(1-x-[1-3x+3x^2-x^3])^{1/3}=(2x-3x^2+x^3)^{1/3}$$

However, more directly, you can check that the function is "anti-symmetric" about $x=1/2$ over the interval $[1/3, 1]$. For each x in the interval, there is a corresponding value at $1-x$ that cancels it out:

$$(x-x^3)^{1/3}+\left((1-x)-(1-x)^3\right)^{1/3}=0$$

This means the area above the x-axis and below the x-axis within the interval exactly cancel, giving a net result of 0.

Question30

The value of $\int_{-3}^{3}\sin^7 x\cos^{16} x\, dx$ is MHT CET 2025 (19 Apr Shift 2)

Options:

A. 1

B. 2

C. 0

D. -1

Answer: C

Solution:

$$\int_{-3}^{3}\sin^7 x\cos^{16} x\, dx$$

$\sin x$ is odd $\Rightarrow \sin^7 x$ is odd.

$\cos x$ is even $\Rightarrow \cos^{16} x$ is even.

Odd × even = odd, so the integrand is an odd function.

Integral of an odd function over a symmetric interval $[-a, a]$ is 0. Hence

$$\boxed{0}.$$

Question31

The value of $\int_{1}^{4}\log[x]dx$ where $[x]$ is the greatest integer function less than or equal to x is equal to MHT CET 2025 (19 Apr Shift 1)

Options:

A. log 5

B. log 6

C. log 2

D. log 3

Answer: B

Solution:

$$\int_1^4 \log(\lfloor x \rfloor)\,dx = \int_1^2 \log 1\,dx + \int_2^3 \log 2\,dx + \int_3^4 \log 3\,dx = 0 + \log 2 + \log 3 = \log 6.$$

$\boxed{\log 6}$ (natural log).

Question32

$\int_1^{\mathrm{e}} \frac{\mathrm{e}^x}{x}(1 + x\log x)\mathrm{d}x =$ **MHT CET 2025 (19 Apr Shift 1)**

Options:

A. e^e

B. $\mathrm{e}^{\mathrm{e}} - \mathrm{e}$

C. $e^e + e$

D. e

Answer: A

Solution:

$$\int_1^e \frac{e^x}{x}(1 + x\log x)\,dx = \int_1^e \left(\frac{e^x}{x} + e^x \log x\right)dx = \int_1^e \frac{d}{dx}(e^x \log x)\,dx = \left[e^x \log x\right]_1^e = e^e - 0 = \boxed{e^e}.$$

Question33

If $[x]$ denotes the greatest integer function, then $\int_0^5 x^2[x]\mathrm{d}x =$ MHT CET 2024 (16 May Shift 2)

Options:

A. $\frac{244}{3}$

B. $\frac{316}{3}$

C. $\frac{200}{3}$

D. $\frac{400}{3}$

Answer: D

Solution:

$$\int_0^5 x^2[x]\mathrm{d}x = \int_0^1 x^2[x]\mathrm{d}x + \int_1^2 x^2[x]\mathrm{d}x + \int_2^3 x^2[x]\mathrm{d}x + \int_3^4 x^2[x] + \int_4^5 x^2[x]\mathrm{d}x$$

$$= \int_0^1 x^2(0)\mathrm{d}x + \int_1^2 x^2(1)\mathrm{d}x + \int_2^3 x^2(2)\mathrm{d}x + \int_3^4 x^2(3)\mathrm{d}x + \int_4^5 x^2(4)\mathrm{d}x$$

$$= \left[\frac{x^3}{3}\right]_i^2 + 2\left[\frac{x^3}{3}\right]_2^3 + 3\left[\frac{x^3}{3}\right]_3^4 + 4\left[\frac{x^3}{3}\right]_4^5$$

$$= \frac{1}{3}(8-1) + \frac{2}{3}(27-8) + (64-27) + \frac{4}{3}(125-64)$$

$$= \frac{400}{3}$$

Question34

The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x}\,\mathrm{d}x$ is MHT CET 2024 (16 May Shift 1)

Options:

A. $\frac{\pi}{4}$

B. $\frac{\pi}{8}$

C. $\frac{\pi}{2}$

D. 4π

Answer: A

Solution:

Let $\mathrm{I}=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} \mathrm{~d}x$...(i)

$\therefore \quad \mathrm{I}=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^{-x}} \mathrm{~d}x$...(ii)

$\cdots\left[\because \int_a^b f(x) \mathrm{d}x=\int_a^b f(a+b-x) \mathrm{d}x\right]$

Adding (i) and (ii), we get

$$2\mathrm{I}=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \,\mathrm{d}x$$

$$\Rightarrow 2\mathrm{I}=2 \int^{\frac{\pi}{2}} \sin^2 x \,\mathrm{d}x$$

$$\left[\begin{array}{l}\because \int_{-a}^{a} f(x) \mathrm{d}x=2 \int_0^a f(x) \mathrm{d}x \\ \quad \text{if } f(x) \text{ is an even function}\end{array}\right]$$

$$\begin{aligned} \Rightarrow \mathrm{I} & =\int_0^{\frac{\pi}{2}}\left(\frac{1-\cos 2x}{2}\right) \mathrm{d}x \\ & =\frac{1}{2}\left[x-\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}} \\ & =\frac{1}{2}\left(\frac{\pi}{2}-0\right)=\frac{\pi}{4}\end{aligned}$$

Question35

$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{1+\cos x}}{(1-\cos x)^{\frac{5}{2}}} dx=$ **MHT CET 2024 (15 May Shift 2)**

Options:

A. $\frac{1}{2}$

B. $\frac{-1}{2}$

C. $\frac{3}{2}$

D. $\frac{-3}{2}$

Answer: C

Solution:

$$I=\int_{\pi/3}^{\pi/2} \frac{\sqrt{1+\cos x}}{(1-\cos x)^{\frac{5}{2}}} dx$$

Let $\quad =\int_{\pi/3}^{\pi/2} \frac{\sqrt{1+\cos x}}{(1-\cos x)^{\frac{5}{2}}} \times \frac{\sqrt{1-\cos x}}{\sqrt{1-\cos x}} dx$

$$=\int_{\pi/3}^{\pi/2} \frac{\sin x}{(1-\cos x)^3} dx$$

$$\begin{aligned} & \text{Put } 1-\cos x=\mathrm{t} \\ & \Rightarrow \sin x \,\mathrm{d}x=\mathrm{dt}\end{aligned}$$

$\therefore \quad \mathrm{I}=\int_{1/2}^1 \frac{\mathrm{dt}}{\mathrm{t}^3}=\left[\frac{\mathrm{t}^{-2}}{-2}\right]_{1/2}^1=\frac{3}{2}$

Question36

$\int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{(1+\tan x)(2+\tan x)} dx =$ **MHT CET 2024 (15 May Shift 2)**

Options:

A. $\log\left(\frac{3}{4}\right)$

B. $\frac{1}{3}\log\left(\frac{4}{3}\right)$

C. $\log\left(\frac{4}{3}\right)$

D. $\frac{1}{4}\log\left(\frac{3}{4}\right)$

Answer: C

Solution:

Put $1+\tan x = \mathrm{t} \Rightarrow \sec^2 x\,\mathrm{d}x = \mathrm{dt}$

When $x = 0, \mathrm{t} = 1$ and when $x = \frac{\pi}{4}, \mathrm{t} = 2$

$$\begin{aligned} \therefore \quad & \int_0^{\pi/4} \frac{\sec^2 x}{(1+\tan x)(2+\tan x)} \mathrm{d}x \\ & = \int_1^2 \frac{\mathrm{dt}}{\mathrm{t}(1+\mathrm{t})} \\ & = \int_1^2 \frac{\mathrm{dt}}{\mathrm{t}} - \int_1^2 \frac{\mathrm{dt}}{1+\mathrm{t}} \\ & = \left[\log \mathrm{t} - \log_{(1+\mathrm{t})}(1)\right]_1^2 \\ & = \log_e 2 - \log_e 3 + \log_e 2 \\ & = \log_e\left(\frac{4}{3}\right) \end{aligned}$$

Question37

The value of integral $\int_{-2}^{0} \left(x^3+3x^2+3x+5+(x+1)\cos(x+1)\right) dx$ is equal to MHT CET 2024 (15 May Shift 1)

Options:

A. 0

B. 6

C. 4

D. 8

Answer: D

Solution:

Let $$I=\int_{-2}^{0}\left[x^3+3x^2+3x+5+(x+1)\cos(x+1)\right]dx$$
$$=\int_{-2}^{0}\left[(x+1)^3+4+(x+1)\cos(x+1)\right]dx$$

Put $x+1=t \Rightarrow dx=dt$

$\therefore \quad I=\int_{-1}^{1}\left(t^3+4+t\cos t\right)dt$

Since t^3 and $t\cos t$ are odd functions.

$\therefore \quad I=\int_{-1}^{1}4dt=4[t]_{-1}^{1}=8$

Question38

If $I=\int_0^{\frac{\pi}{4}}\log(1+\tan x)dx$, then value of I is MHT CET 2024 (11 May Shift 2)

Options:

A. $\frac{\pi}{16}\log 2$

B. $\frac{\pi}{2}\log 2$

C. $\frac{\pi}{8}\log 2$

D. $\frac{\pi}{4}\log 2$

Answer: C

Solution:

$$\text{Let } I=\int_0^{\frac{\pi}{4}}\log(1+\tan\theta)d\theta$$
$$=\int_0^{\frac{\pi}{4}}\log\left[1+\tan\left(\frac{\pi}{4}-\theta\right)\right]d\theta$$
$$\ldots\left[\because\int_0^{a}f(x)dx=\int_0^{2}f(a-x)dx\right]$$
$$=\int_0^{\frac{\pi}{4}}\log\left(1+\frac{1-\tan\theta}{1+\tan\theta}\right)d\theta$$
$$=\int_0^{\frac{\pi}{4}}\log 2\,d\theta-\int_0^{\frac{\pi}{4}}\log(1+\tan\theta)d\theta$$
$$\therefore\quad 2I=\int_0^{\frac{\pi}{4}}\log 2\,d\theta\Rightarrow I=\frac{\log 2}{2}[\theta]_0^{\pi/4}=\frac{\pi}{8}\log 2$$

Question39

$\int_{0.2}^{3.5}[x]dx=$(where $[x]=$ greatest integer not greater than x) MHT CET 2024 (11 May Shift 1)

Options:

A. 4

B. 4.2

C. 4.5

D. 4.4

Answer: C

Solution:

$$\begin{aligned}\int_{0.2}^{3.5}[x]\mathrm{d}x &= \int_{0.2}^{1}(0)\mathrm{d}x+\int_{1}^{2}(1)\mathrm{d}x+\int_{2}^{3}2\ \mathrm{d}x+\int_{3}^{3.5}3\ \mathrm{d}x\\ &= 0+[x]_1^2+2[x]_2^3+3[x]_3^{3.5}\\ &= 0+(2-1)+2(3-2)+3(3.5-3)\\ &= 0+1+2+1.5\\ &= 4.5\end{aligned}$$

Question40

$\int_0^{\frac{\pi}{4}}\log\left(\frac{\sin x+\cos x}{\cos x}\right)dx=$ **MHT CET 2024 (10 May Shift 2)**

Options:

A. $\frac{\pi}{2}\log 2$

B. $\frac{\pi}{4}\log 2$

C. $\frac{\pi}{6}\log 2$

D. $\frac{\pi}{8}\log 2$

Answer: D

Solution:

$$I=\int_0^{\frac{\pi}{4}}\log\left(\frac{\sin x+\cos x}{\cos x}\right)dx$$

Let

$$=\int_0^{\frac{\pi}{4}}\log(1+\tan\theta)\mathrm{d}\theta$$

$$=\int_0^{\frac{\pi}{4}}\log\left[1+\tan\left(\frac{\pi}{4}-\theta\right)\right]\mathrm{d}\theta$$

$$\cdots\left[\because\int_0^a f(x)\mathrm{d}x=\int_0^a \mathrm{f}(\mathrm{a}-x)\mathrm{d}x\right]$$

$$=\int_0^{\frac{\pi}{4}}\log\left(1+\frac{1-\tan\theta}{1+\tan\theta}\right)d\theta$$

$$=\int_0^{\frac{\pi}{4}}\log 2d\theta-\int_0^{\frac{\pi}{4}}\log(1+\tan\theta)d\theta$$

$$\therefore\quad 2I=\int_0^{\frac{\pi}{4}}\log 2d\theta\Rightarrow I=\frac{\log 2}{2}[\theta]_0^{\pi/4}=\frac{\pi}{8}\log 2$$

Question41

$\int_0^2\frac{x-\mathrm{a}}{x+\mathrm{a}}\mathrm{d}x=$ **MHT CET 2024 (10 May Shift 2)**

Options:

A. $\mathrm{a}-2\mathrm{a}\log 2$

B. $a-a\log 2$

C. $\mathrm{a}+2\mathrm{a}\log 2$

D. $a+a\log 2$

Answer: A

Solution:

$$\begin{aligned}
&\text{Let } \mathrm{I}=\int_0^{\mathrm{a}} \frac{x-\mathrm{a}}{x+\mathrm{a}}\,\mathrm{d}x \\
&\text{Let } x+\mathrm{a}=\mathrm{t} \\
&\Rightarrow x=\mathrm{t}-\mathrm{a} \\
&\text{If } x=0, \text{ then } \mathrm{t}=\mathrm{a} \\
&\text{If } x=\mathrm{a}, \text{ then } \mathrm{t}=2\mathrm{a} \\
&\therefore \quad \mathrm{d}x=\mathrm{dt} \\
&\therefore \quad I=\int_a^{2a} \frac{t-2a}{t} dt \\
&=\int_a^{2a} 1 dt-2a\int_a^{2a} \frac{1}{t} dt=[t]_a^{2a}-2a[\log t]_a^{2a} \\
&=\mathrm{a}-2\mathrm{a}(\log 2\mathrm{a}-\log \mathrm{a}) \\
&=a-2a\log 2
\end{aligned}$$

Question42

$\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}}(\sin x)^{-4}\,\mathrm{d}x$ **has the value MHT CET 2024 (10 May Shift 2)**

Options:

A. $\frac{-3}{2}$

B. $\frac{3}{2}$

C. $\frac{-8}{3}$

D. $\frac{8}{3}$

Answer: C

Solution:

Let

$$
\begin{aligned}
I &= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (\sin x)^{-4}\, dx \\
&= 2\int_{0}^{\frac{\pi}{4}} (\sin x)^{-4}\, dx \\
&\quad \ldots [\because (\sin x)^{-4} \text{ is an even function }] \\
&= 2\int_{0}^{\frac{\pi}{4}} \frac{1}{\sin^4 x}\, dx \\
&= 2\int_{0}^{\frac{\pi}{4}} \frac{\sec^4 x}{\tan^4 x}\, dx \\
&= 2\int_{0}^{\frac{\pi}{4}} \frac{(1+\tan^2 x)\sec^2 x}{\tan^4 x}\, dx
\end{aligned}
$$

Let $\tan x = t$ when $x = 0$, we get $t = 0$

when $x = \frac{\pi}{4}$, we get $t = 1$

Also, $\sec^2 x\, dx = dt$

$$
\begin{aligned}
\therefore \quad I &= 2\int_{0}^{1} \frac{1+t^2}{t^4} dt \\
&= 2\left[\int_{0}^{1} \frac{1}{t^4} dt + \int_{0}^{1} \frac{1}{t^2} dt\right] \\
&= 2\left[\frac{[t^{-3}]_0^1}{-3} + \frac{[t^{-1}]_0^1}{-1}\right] \\
&= \frac{-8}{3}
\end{aligned}
$$

Question43

$\int_0^{\frac{\pi}{2}} |\sin x - \cos x| dx$ **has the value MHT CET 2024 (10 May Shift 1)**

Options:

A. $2\sqrt{2} + 1$

B. $2(\sqrt{2} + 1)$

C. $2(\sqrt{2} - 1)$

D. $2\sqrt{2} - 1$

Answer: C

Solution:

$$I=\int_0^{\frac{\pi}{2}}|\sin x-\cos x|dx$$
$$=\int_0^{\frac{\pi}{4}}-(\sin x-\cos x)dx+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(\sin x-\cos x)dx$$

$$\ldots\left\{\begin{aligned}\mathrm{f}(x)&=\mathrm{f}(-x) & 0\le x\le\frac{\pi}{4}\\ &=\mathrm{f}(x) & \frac{\pi}{4}\le x\le\frac{\pi}{2}\end{aligned}\right\}$$

$$=\int_0^{\frac{\pi}{4}}(\cos x-\sin x)dx+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(\sin x-\cos x)dx$$
$$=(\sin x+\cos x)_0^{\frac{\pi}{4}}+(-\cos x-\sin x)_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$
$$=\left[\sin\frac{\pi}{4}+\cos\frac{\pi}{4}-\sin 0-\cos 0\right]$$

$$-\left[\cos\tfrac{\pi}{2}+\sin\tfrac{\pi}{2}-\cos\tfrac{\pi}{4}-\sin\tfrac{\pi}{4}\right]$$

$$=\left[\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}-0-1\right]-\left[0+1-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right]$$
$$=\left[\frac{2}{\sqrt{2}}-1\right]-\left[1-\frac{2}{\sqrt{2}}\right]$$
$$=\frac{4}{\sqrt{2}}-2$$
$$=\frac{4\sqrt{2}}{2}-2$$
$$=2\sqrt{2}-2$$
$$=2(\sqrt{2}-1)$$

Question44

The integral $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\frac{dx}{\sin 2x\left(\tan^5 x+\cot^5 x\right)}$ is equal to MHT CET 2024 (09 May Shift 2)

Options:

A. $\frac{1}{5}\left(\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{3\sqrt{3}}\right)\right)$

B. $\frac{1}{10}\left(\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right)$

C. $\frac{1}{20}\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)$

D. $\frac{\pi}{40}$

Answer: B

Solution:

$$\text{Let } I=\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{dx}{\sin 2x\left(\tan^5 x+\cot^5 x\right)}$$
$$=\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\mathrm{d}x}{2 \sin x \cos x\left(\tan^5 x+\frac{1}{\tan^5 x}\right)}$$
$$=\frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\sec^2 x}{\tan x\left(\frac{\tan^{10} x+1}{\tan^5 x}\right)} \mathrm{d}x$$
$$=\frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\tan^4 x \sec^2 x}{\tan^{10} x+1} \mathrm{~d}x$$

Put $\tan^5 x=\mathrm{t} \Rightarrow 5 \tan^4 x \sec^2 x \mathrm{~d}x=\mathrm{dt}$

$$\therefore \quad \mathrm{I}=\frac{1}{2} \int_{\frac{1}{9\sqrt{3}}}^{1} \frac{\frac{\mathrm{dt}}{5}}{\mathrm{t}^2+1}$$
$$=\frac{1}{10}\left[\tan^{-1} \mathrm{t}\right]_{\frac{1}{9\sqrt{3}}}^{1}$$
$$=\frac{1}{10}\left[\tan^{-1} 1-\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right]$$
$$=\frac{1}{10}\left[\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right]$$

Question45

The value of $I=I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \cos x}{1+e^{-x}} \mathrm{~d}x$ is equal to MHT CET 2024 (09 May Shift 1)

Options:

A. $\frac{\pi^2}{4}-2$

B. $\frac{\pi^2}{4}+2$

C. $\pi^2-\mathrm{e}^{\frac{\pi}{2}}$

D. $\pi^2+\mathrm{e}^{\frac{\pi}{2}}$

Answer: A

Solution:

$$I=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \cos x}{1+e^{-x}}\, \mathrm{d}x \ldots (i)$$

$$I=\frac{\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\left(\frac{-\pi}{2}+\frac{\pi}{2}-x\right)^2 \cdot \cos\left(-\frac{\pi}{2}+\frac{\pi}{2}-x\right)}{1+e^{\left(-\frac{\pi}{2}+\frac{\pi}{2}+x\right)}}}{\ldots\left[\int_a^b f(x) dx=\int_a^b f(a+b-x) dx\right]}$$

$$=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \cos x}{1+\mathrm{e}^x}$$

Adding equation (i) and (ii), we get

$$2\mathrm{I}=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\left(\frac{x^2 \cos x}{1+\mathrm{e}^x}+\frac{x^2 \cos x}{1+\mathrm{e}^{-x}}\right) \mathrm{d}x$$

$$2\mathrm{I}=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} x^2 \cos x\left[\frac{1}{1+\mathrm{e}^x}+\frac{1}{1+\mathrm{e}^{-x}}\right] \mathrm{d}x$$

$$2\mathrm{I}=\int_{\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \cos x\left[\frac{1}{1+\mathrm{e}^x}+\frac{\mathrm{e}^x}{\mathrm{e}^x+1}\right] \mathrm{d}x$$

$$\therefore \quad 2\mathrm{I}=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \cdot \cos x \, \mathrm{d}x$$

$$\therefore \quad 2\mathrm{I}=2 \int_0^{\frac{\pi}{2}} x^2 \cos x \, \mathrm{d}x$$

$$\therefore \left[\begin{array}{l}\int_{-\mathrm{a}}^{\mathrm{a}} \mathrm{f}(x)=2 \int_0^{\mathrm{a}} \mathrm{f}(x) \mathrm{d}x \\ \text{if } \mathrm{f}(x) \text{ is even function}\end{array}\right]$$

$$\begin{aligned} \therefore \quad I & =\int_0^{\frac{\pi}{2}} x^2 \cos x \cdot dx \\ & =\left[x^2 \cdot \sin x-2 \int x \cdot \sin x \, \mathrm{d}x\right]_0^{\frac{\pi}{2}} \\ & =\left[x^2 \sin x-2\left(-x \cos x+\int \cos x \, \mathrm{d}x\right)\right]_0^{\frac{\pi}{2}} \end{aligned}$$

$$\begin{aligned} & =\left[x^2 \sin x+2 x \cos x-2 \sin x\right]_0^{\frac{\pi}{2}} \\ & =\left(\frac{\pi^2}{4}-2 \sin \frac{\pi}{2}-0+0-0\right) \\ & =\frac{\pi^2}{4}-2 \end{aligned}$$

Question46

The value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{1}{\sin 2x\left(\tan^5 x+\cot^5 x\right)} \mathrm{dx}$ **is MHT CET 2024 (04 May Shift 2)**

Options:

A. $\frac{1}{5}\left(\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{3\sqrt{3}}\right)\right)$

B. $\frac{1}{2}\left(\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right)$

C. $\frac{1}{10}\left(\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right)$

D. $\frac{1}{10}\left(\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{3\sqrt{3}}\right)\right)$

Answer: C

Solution:

$$I=\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{1}{\sin 2x\left(\tan^5 x+\cot^5 x\right)} dx$$
$$=\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{1}{\frac{2\tan x}{1+\tan^2 x}\left(\tan^5 x+\frac{1}{\tan^5 x}\right)} dx$$
$$=\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\left(1+\tan^2 x\right)}{2\tan x\left(\tan^5 x+\frac{1}{\tan^5 x}\right)} dx$$

$$I=\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\sec^2 x}{2\tan x\left(\tan^5 x+\frac{1}{\tan^5 x}\right)} dx$$

Let $\tan x=\mathrm{t} \sec^2 x\, \mathrm{d}x=\mathrm{dt}$ when $x=\frac{\pi}{6}, \mathrm{t}=\frac{1}{\sqrt{3}}$ when $x=\frac{\pi}{4}, t=1$

$$\therefore \quad I=\int_{\frac{1}{\sqrt{3}}}^{1} \frac{dt}{2t\left(t^5+\frac{1}{t^5}\right)}$$
$$I=\int_{\frac{1}{\sqrt{3}}}^{1} \frac{t^4}{2\left(t^{10}+1\right)} dt$$
Let $t^5=u$
$$\therefore \quad 5t^4 dt=du$$

When $\mathrm{t}=\frac{1}{\sqrt{3}}, \mathrm{u}=3^{\frac{-5}{2}}$

When $\mathrm{t}=1, \mathrm{u}=1$

$$\therefore \quad I=\frac{1}{10} \int_{\frac{-5}{32}}^{1} \frac{du}{\left(u^2+1\right)}$$
$$=\frac{1}{10}\left(\tan^{-1} u\right)_{\frac{-5}{2}}^{1}$$
$$=\frac{1}{10}\left[\tan^{-1} 1-\tan^{-1}\left(3^{\frac{-5}{2}}\right)\right]$$
$$=\frac{1}{10}\left[\frac{\pi}{4}-\tan^{-1} \frac{1}{\left(3^{\frac{5}{2}}\right)}\right]$$
$$\therefore \quad I=\frac{1}{10}\left[\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right]$$

Question47

The integral $\int_{\frac{-1}{2}}^{\frac{1}{2}}\left([x]+\log_c\left(\frac{1+x}{1-x}\right)\right) \mathrm{d}x$, where $[x]$ represent greatest integer function, equals MHT CET 2024 (04 May Shift 1)

Options:

A. $-\frac{1}{2}$

B. $\log_e\left(\frac{1}{2}\right)$

C. $\frac{1}{2}$

D. $2\log_{\mathrm{e}}\left(\frac{1}{2}\right)$

Answer: A

Solution:

Let $I=\int_{\frac{-1}{2}}^{\frac{1}{2}}\left([x]+\log_e\left(\frac{1+x}{1-x}\right)\right)dx$

$=\int_{\frac{-1}{2}}^{0}[x]dx+\int_{0}^{\frac{1}{2}}[x]dx+\int_{\frac{-1}{2}}^{\frac{1}{2}}\log_e\left(\frac{1+x}{1-x}\right)dx$

Let $g(x)=\log\left(\frac{1+x}{1-x}\right)$

$g(-x)=\log\left(\frac{1-x}{1+x}\right)=-\log\left(\frac{1+x}{1-x}\right)=-g(x)$

$\therefore$ $g(x)$ is a odd function.

$\therefore$ $\int_{\frac{-1}{2}}^{\frac{1}{2}}g(x)dx=0$

$$\begin{aligned}\therefore\quad I&=\int_{\frac{-1}{2}}^{0}(-1)dx+\int_{0}^{\frac{1}{2}}(0)dx+0\\&=[-x]_{\frac{-1}{2}}^{0}+0\\&=\frac{-1}{2}\end{aligned}$$

Question48

The value of the integral $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\left(x^2+\log\frac{\pi-x}{\pi+x}\right)\cos x dx$ is equal to MHT CET 2024 (03 May Shift 2)

Options:

A. 0

B. $\frac{\pi^2}{2}-4$

C. $\frac{\pi^2}{2}$

D. $\frac{\pi^2}{2}+4$

Answer: B

Solution:

Let $\mathrm{I}=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left[x^2+\log\left(\frac{\pi-x}{\pi+x}\right)\right]\cos x\,\mathrm{d}x$

$\mathrm{I}=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} x^2\cos x\,\mathrm{d}x+\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\log\left(\frac{\pi-x}{\pi+x}\right)\cos x\,\mathrm{d}x$

Let $\mathrm{I}=\mathrm{I}_1+\mathrm{I}_2$

Where $\mathrm{I}_1=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} x^2\cos x\,\mathrm{d}x$ and

$\mathrm{I}_2=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\log\left(\frac{\pi-x}{\pi+x}\right)\cos x\,\mathrm{d}x$

Consider

$$I_1=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} x^2\cos x\cdot dx$$
$$=2\int_0^{\frac{\pi}{2}} x^2\cos x dx$$

$\ldots\left[x^2\cos x\right.$ is an even function $\left.\right]$

$$=2\left[x^2\cdot\int\cos x\,\mathrm{d}x-\int\frac{\mathrm{d}}{\mathrm{d}x}\left(x^2\right)\left(\int\cos x\,\mathrm{d}x\right)\mathrm{d}x\right]_0^{\frac{\pi}{2}}$$

$$=2\left[x^2\cdot\sin x-\int 2x\cdot\sin x dx\right]_0^{\frac{\pi}{2}}$$
$$=2\left[x^2\sin x-2\int x\cdot\sin x dx\right]_0^{\frac{\pi}{2}}$$

$$=2\left[x^2\sin x-2\left(x(-\cos x)-\int(-\cos x)\mathrm{d}x\right)\right]_0^{\frac{\pi}{2}}$$

$$=2\left[x^2\sin x+2x\cos x-2\sin x\right]_0^{\frac{\pi}{2}}$$
$$=2\left[\frac{\pi^2}{4}\sin\frac{\pi}{2}+2\times\frac{\pi}{2}\cos\frac{\pi}{2}-2\sin\frac{\pi}{2}\right.$$

$$\left.-0^2\sin 0-2\times 0\times\cos 0+2\sin 0\right]$$

$$=2\left[\frac{\pi^2}{4}-2-0-0-0\right]$$
$$=2\left[\frac{\pi^2}{4}-2\right]=\frac{\pi^2}{2}-4$$

Consider

$$\mathrm{I}_2=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\log\left(\frac{\pi-x}{\pi+x}\right)\cos x\,\mathrm{d}x$$
$$\therefore\quad \mathrm{I}_2=0\quad\ldots\left[\log\left(\frac{\pi-x}{\pi+x}\right)\cos x\text{ is an odd function}\right]$$
$$\therefore\quad \mathrm{I}=\mathrm{I}_1+\mathrm{I}_2$$
$$\Rightarrow I=\frac{\pi^2}{2}-4$$

Question49

If $\int_0^{\frac{\pi}{3}}\frac{\tan\theta}{\sqrt{2\mathrm{k}\sec\theta}}\mathrm{d}\theta=1-\frac{1}{\sqrt{2}},(k>0)$, then the value of k is MHT CET 2024 (03 May Shift 1)

Options:

A. 2

B. 1

C. $\frac{1}{2}$

D. 4

Answer: A

Solution:

$$\text{Let } \mathrm{I}=\int_0^{\frac{\pi}{3}} \frac{\tan \theta}{\sqrt{2 \mathrm{k} \sec \theta}} \mathrm{d} \theta$$
$$=\frac{1}{\sqrt{2 \mathrm{k}}} \int_0^{\frac{\pi}{3}} \frac{\sin \theta}{\cos \theta} \times \sqrt{\cos \theta} \mathrm{d} \theta$$
$$=\frac{1}{\sqrt{2 \mathrm{k}}} \int_0^{\frac{\pi}{3}} \frac{\sin \theta}{\sqrt{\cos \theta}} \mathrm{d} \theta$$

$$\therefore \begin{aligned} & \text{Let } \cos \theta=\mathrm{t} \\ & \sin \theta \mathrm{d} \theta=-\mathrm{dt} \end{aligned}$$
$$\therefore \quad \text{when } \theta=\frac{\pi}{3}, \mathrm{t}=\frac{1}{2}$$
$$\text{when } \theta=0, \mathrm{t}=1$$
$$\therefore \quad I=\frac{-1}{\sqrt{2 \mathrm{k}}} \int_1^{\frac{1}{2}} \frac{1}{\sqrt{\mathrm{t}}} \mathrm{dt}$$
$$=\frac{-1}{\sqrt{2 \mathrm{k}}} \frac{\left.\mathrm{t}^{\frac{1}{2}}\right]_1^{\frac{1}{2}}}{\frac{1}{2}}$$

$$=\frac{-\sqrt{2}}{\sqrt{\mathrm{k}}}\left(\frac{1}{\sqrt{2}}-1\right)=\frac{\sqrt{2}}{\sqrt{\mathrm{k}}}\left(1-\frac{1}{\sqrt{2}}\right)$$
Given that $I=\left(1-\frac{1}{\sqrt{2}}\right)$

$$\Rightarrow \frac{\sqrt{2}}{\sqrt{\mathrm{k}}}=1 \Rightarrow \mathrm{k}=2$$

Question50

The value of $\mathrm{I}=\int_{\sqrt{\log_e 2}}^{\sqrt{\log_e 3}} \frac{x \sin x^2}{\sin x^2+\sin \left(\log_e 6-x^2\right)} \mathrm{d} x$ is MHT CET 2024 (02 May Shift 2)

Options:

A. $\frac{1}{4} \log_e \frac{3}{2}$

B. $\frac{1}{2} \log_e \frac{3}{2}$

C. $\log_e \frac{3}{2}$

D. $\frac{1}{6} \log_e \frac{3}{2}$

Answer: A

Solution:

Let $I=\int_{\sqrt{\log 2}}^{\sqrt{\log 3}} \frac{x \sin x^2}{\sin x^2+\sin(\log 6-x^2)} dx$

$$\text{Put } x^2=t \Rightarrow 2x\,dx=dt$$

$$\therefore \quad I=\frac{1}{2}\int_{\log_2}^{\log_3} \frac{\sin t}{\sin t+\sin(\log 6-t)} dt \ldots (i)$$

$$\therefore \quad I=\frac{1}{2}\int_{\log 2}^{\log_2 3} \frac{\sin(\log 6-t)}{\sin(\log 6-t)+\sin t} dt \ldots (ii)$$

$$\cdots\left[\because \int_a^b f(x)dx=\int_a^b f(a+b-x)dx\right]$$

Adding (i) and (ii), we get

$$2I=\frac{1}{2}\int_{\log 2}^{\log 3} dt=\frac{1}{2}(\log 3-\log 2)=\frac{1}{2}\log\left(\frac{3}{2}\right)$$
$$\Rightarrow I=\frac{1}{4}\log\left(\frac{3}{2}\right)$$

Question51

Let f and g be continuous functions on $[0, a]$ such that $f(x)=f(a-x)$ and $g(x)+g(a-x)=4$, then $\int_0^a f(x)g(x)dx$ is equal to MHT CET 2024 (02 May Shift 1)

Options:

A. $4\int_0^a f(x)dx$

B. $\int_0^a f(x)dx$

C. $2\int_0^a f(x)dx$

D. $-3\int_0^a f(x)dx$

Answer: C

Solution:

Given,

$$f(x)=f(a-x)$$
$$g(x)+g(a-x)=4$$
$$\text{Let } I=\int_0^a f(x)g(x)dx$$
$$=\int_0^a f(a-x)\cdot g(a-x)dx$$
$$=\int_0^a f(x)\cdot[(4-g(x)]dx$$
$$=4\int_0^a f(x)dx-\int_0^a f(x)g(x)dx$$
$$I=4\int_0^a f(x)dx-I$$
$$2I=4\int_0^a f(x)dx$$
$$\therefore \quad I=2\int_0^a f(x)dx$$

Question52

$\int \frac{dx}{3-2\cos 2x} = \frac{\tan^{-1}(f(x))}{\sqrt{5}} + c$, **(where c is constant of integration), then $f(\pi/4)$ has the value MHT CET 2024 (02 May Shift 1)**

Options:

A. $-\sqrt{5}$

B. $\sqrt{5}$

C. $\frac{2}{\sqrt{5}}$

D. $\frac{1}{\sqrt{5}}$

Answer: B

Solution:

$$\text{Let } I = \int \frac{dx}{3-2\cos 2x}$$

Put $\tan x = t$

$$\therefore \quad x = \tan^{-1} t$$

$$\therefore \quad dx = \frac{dt}{1+t^2}$$

$$\cos 2x = \frac{1-t^2}{1+t^2}$$

$$\therefore \quad I = \int \frac{\frac{dt}{1+t^2}}{3-2\left(\frac{1-t^2}{1+t^2}\right)}$$

$$= \int \frac{dt}{3(1+t^2)-2(1-t^2)}$$

$$= \int \frac{dt}{3+3t^2-2+2t^2}$$

$$= \int \frac{dt}{(\sqrt{5}t)^2+(1)^2}$$

$$= \frac{1}{\sqrt{5}}\tan^{-1}(\sqrt{5}t)+c$$

$$\therefore \quad I = \frac{1}{\sqrt{5}}\tan^{-1}\sqrt{5}\tan x + c$$

Comparing with $\frac{\tan^{-1} f(x)}{\sqrt{5}}$, we get

$$f(x) = \sqrt{5}\tan x$$

$$\therefore \quad f\left(\frac{\pi}{4}\right) = \sqrt{5}\tan\frac{\pi}{4} = \sqrt{5}$$

Question53

If $I_n = \int_0^{\frac{\pi}{4}} \tan^n \theta d\theta$, then $I_{12} + I_{10} =$ MHT CET 2023 (14 May Shift 2)

Options:

A. $\frac{1}{8}$

B. $\frac{1}{12}$

C. $\frac{1}{11}$

D. $\frac{1}{10}$

Answer: C

Solution:

$$\int_0^{\frac{\pi}{4}}\left(\tan^{n} x+\tan^{n-2} x\right) dx=\frac{1}{n-1}$$

$$\therefore \quad I_{12}+I_{10}=\int_0^{\frac{\pi}{4}}\left(\tan^{12} \theta+\tan^{10} \theta\right) d\theta$$

$$=\frac{1}{12-1}$$

$$=\frac{1}{11}$$

Question54

The integral $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^{\frac{2}{3}} x \operatorname{cosec}^{\frac{4}{3}} x dx$ is equal to MHT CET 2023 (14 May Shift 1)

Options:

A. $3^{\frac{5}{6}}-3^{\frac{2}{3}}$

B. $3^{\frac{7}{6}}-3^{\frac{5}{6}}$

C. $3^{\frac{5}{3}}-3^{\frac{1}{3}}$

D. $3^{\frac{4}{3}}-3^{\frac{1}{3}}$

Answer: B

Solution:

$$\text{Let } I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^{\frac{2}{3}} x \operatorname{cosec}^{\frac{4}{3}} x \, dx$$

$$=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{\cos^{\frac{2}{3}} x \cdot \sin^{\frac{4}{3}} x}$$

$$=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{\frac{\sin^{\frac{4}{3}} x}{\cos^{\frac{4}{3}} x} \cdot \cos^2 x}$$

$$=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sec^2 x}{\tan^{\frac{4}{3}} x} \, dx$$

$$\text{Put } \tan x=t$$
$$\Rightarrow \sec^2 x \, dx=dt$$
$$\therefore \quad I=\int_{\frac{1}{\sqrt{3}}} t^{\frac{\sqrt{3}}{3}} \frac{dt}{x^{-1}}$$
$$=\left[-3t^{-\frac{1}{3}}\right]_{\frac{1}{\sqrt{3}}}^{\sqrt{3}}$$
$$=-3\left[(\sqrt{3})^{\frac{-1}{3}}-\left(\frac{1}{\sqrt{3}}\right)^{\frac{-1}{3}}\right]$$
$$=-3\left(3^{-\frac{1}{6}}-3^{\frac{1}{6}}\right)$$
$$=3^{\frac{7}{6}}-3^{\frac{5}{6}}$$

Question55

The integral $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^{\frac{2}{3}} x \operatorname{cosec}^{\frac{4}{3}} x \, dx$ is equal to MHT CET 2023 (13 May Shift 2)

Options:

A. $3^{\frac{5}{6}} - 3^{\frac{2}{3}}$

B. $3^{\frac{7}{6}} - 3^{\frac{5}{6}}$

C. $3^{\frac{5}{3}} - 3^{\frac{1}{3}}$

D. $3^{\frac{4}{3}} - 3^{\frac{1}{3}}$

Answer: B

Solution:

Let $\mathrm{I} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^{\frac{2}{3}} x \operatorname{cosec}^{\frac{4}{3}} x \, dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{\cos^{\frac{2}{3}} x \cdot \sin^{\frac{4}{3}} x}$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{\frac{\sin^{\frac{4}{3}} x}{\cos^{\frac{4}{3}} x} \cdot \cos^2 x}$$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sec^2 x}{\tan^{\frac{4}{3}} x} \, dx$

Put $\tan x = \mathrm{t} \Rightarrow \sec^2 x \, dx = dt$

$\therefore \quad I = \int_{\frac{1}{\sqrt{3}}}^{t^{\frac{\sqrt{3}}{3}}} \frac{dt}{\frac{4}{3}} = \left[-3t^{-\frac{1}{3}}\right]_{\frac{1}{\sqrt{3}}}^{\sqrt{3}}$

$= -3\left[(\sqrt{3})^{\frac{-1}{3}} - \left(\frac{1}{\sqrt{3}}\right)^{\frac{-1}{3}}\right]$

$= -3\left(3^{-\frac{1}{6}} - 3^{\frac{1}{6}}\right)$

$= 3^{\frac{7}{6}} - 3^{\frac{5}{6}}$

Question56

If $f(x) = \begin{cases} e^{\cos x} \sin x & , \text{ for } |x| \le 2 \\ 2, & \text{otherwise} \end{cases}$, then $\int_{-2}^{3} f(x) dx$ is equal to MHT CET 2023 (13 May Shift 1)

Options:

A. 0

B. 2

C. 1

D. 3

Answer: B

Solution:

$$\int_{-2}^{3} \mathrm{f}(x) \mathrm{d}x = \int_{-2}^{2} \mathrm{f}(x) \mathrm{d}x + \int_{2}^{3} \mathrm{f}(x) \mathrm{d}x$$
$$= \int_{-2}^{2} \mathrm{e}^{\cos x} \sin x \, \mathrm{d}x + \int_{2}^{3} 2 \, \mathrm{d}x$$

Since $\mathrm{e}^{\cos x} \sin x$ is an odd function.

$$\therefore \quad \int_{-2}^{3} \mathrm{f}(x) \mathrm{d}x = 0 + 2(3-2) = 2$$

Question57

The value of $\int_0^\pi \left|\sin x - \frac{2x}{\pi}\right| \mathrm{d}x$ is MHT CET 2023 (12 May Shift 2)

Options:

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. π

D. 2π

Answer: B

Solution:

$$\int_0^\pi \left|\sin x - \frac{2x}{\pi}\right| \mathrm{d}x$$
$$= \int_0^{\frac{\pi}{2}} \left(\sin x - \frac{2x}{\pi}\right) \mathrm{d}x + \int_{\frac{\pi}{2}}^{\pi} \left(\frac{2x}{\pi} - \sin x\right) \mathrm{d}x$$
$$= [-\cos x]_0^{\frac{\pi}{2}} - \left[\frac{x^2}{\pi}\right]_0^{\frac{\pi}{2}} + \left[\frac{x^2}{\pi}\right]_{\frac{\pi}{2}}^{\pi} + [\cos x]_{\frac{\pi}{2}}^{\pi}$$
$$= (0+1) - \left(\frac{\pi}{4} - 0\right) + \left(\pi - \frac{\pi}{4}\right) + (-1-0)$$
$$= \frac{\pi}{2}$$

Question58

$$\int_0^4 |2x-5| \mathrm{d}x =$$

MHT CET 2023 (12 May Shift 1)

Options:

A. $\frac{13}{2}$

B. $\frac{15}{2}$

C. $\frac{17}{4}$

D. $\frac{17}{2}$

Answer: D

Solution:

$$\begin{aligned}\int_0^4|2x-5|\mathrm{d}x &= \int_0^{\frac{5}{2}}(5-2x)\mathrm{d}x+\int_{\frac{5}{2}}^4(2x-5)\mathrm{d}x\\ &= \left[5x-x^2\right]_0^{\frac{5}{2}}+\left[x^2-5x\right]_{\frac{5}{2}}^4\\ &= \frac{25}{4}+\frac{9}{4}\\ &= \frac{34}{4}\\ &= \frac{17}{2}\end{aligned}$$

Question59

$\int_0^\pi \frac{\mathrm{d}x}{4+3\cos x}=$ **MHT CET 2023 (11 May Shift 1)**

Options:

A. $\frac{2\pi}{7}$

B. $\frac{\pi}{\sqrt{7}}$

C. $\frac{\pi}{2\sqrt{7}}$

D. $\frac{\pi}{7}$

Answer: B

Solution:

Let $\mathrm{I}=\int_0^\pi \frac{\mathrm{dx}}{4+3\cos x}$

Put $\tan\frac{x}{2}=\mathrm{t}$

$\therefore\quad \mathrm{d}x=\frac{2\mathrm{dt}}{1+\mathrm{t}^2}$ and $\cos x=\frac{1-\mathrm{t}^2}{1+\mathrm{t}^2}$

$$\begin{aligned}\therefore\quad \mathrm{I}=\int_0^\pi\frac{\mathrm{d}x}{4+3\cos x} &= \int_0^\infty\frac{2\mathrm{dt}}{7+\mathrm{t}^2}\\ &= \left[\frac{2}{\sqrt{7}}\tan^{-1}\left(\frac{\mathrm{t}}{\sqrt{7}}\right)\right]_0^\infty\\ &= \frac{2}{\sqrt{7}}\left[\tan^{-1}\infty-0\right]\\ &= \frac{2}{\sqrt{7}}\cdot\frac{\pi}{2}=\frac{\pi}{\sqrt{7}}\end{aligned}$$

Question60

Let $f(x)$ **be positive for all real** x. **If** $I_1 = \int_{1-h}^{h} x f(x(1-x)) dx$ **and** $I_2 = \int_{1-h}^{h} f(x(1-x)) dx$, **where** $(2h-1) > 0$, **then** $\frac{I_1}{I_2}$ **is MHT CET 2023 (11 May Shift 1)**

Options:

A. 2

B. h

C. $\frac{1}{2}$

D. 1

Answer: C

Solution:

$$I_1 = \int_{1-h}^{h} x f(x(1-x)) dx \text{ and}$$

$$I_2 = \int_{1-h}^{h} f(x(1-x)) dx$$

$$I_1 = \int_{1-h}^{h} (1-x) f[(1-x)(1-1+x)] dx$$

$$\therefore \quad \ldots \left[\because \int_a f(x) dx = \int_a^b f(a+b-x) dx\right] I_1 = \int_{1-h}^{h} (1-x) f(x(1-x)) dx$$

$$= \int_{1-h}^{h} f(x(1-x)) dx - \int_{1-h}^{h} x f(x(1-x)) dx$$

$$\Rightarrow I_1 = I_2 - I_1$$

$$\Rightarrow 2I_1 = I_2$$

$$\Rightarrow \frac{I_1}{I_2} = \frac{1}{2}$$

Question61

$\int_{-1}^{3} \left(\cot^{-1}\left(\frac{x}{x^2+1}\right) + \cot^{-1}\left(\frac{x^2+1}{x}\right)\right) dx =$ **MHT CET 2023 (11 May Shift 1)**

Options:

A. $\left(\frac{\pi}{4}\right)$

B. π

C. $\left(\frac{\pi}{2}\right)$

D. (2π)

Answer: D

Solution:

$$\begin{aligned}\text{Let I } &= \int_{-1}^{3}\left(\cot^{-1}\left(\frac{x}{x^2+1}\right)+\cot^{-1}\left(\frac{x^2+1}{x}\right)\right)dx\\ &= \int_{-1}^{3}\left(\tan^{-1}\left(\frac{x^2+1}{x}\right)+\cot^{-1}\left(\frac{x^2+1}{x}\right)\right)dx\\ &= \int_{-1}^{3}\frac{\pi}{2}\,dx \quad \ldots\left[\because \cot^{-1}(x)=\tan^{-1}\left(\frac{1}{x}\right)\right]\\ &= \frac{\pi}{2}[x]_{-1}^{3}\\ &= \frac{\pi}{2}(4)\end{aligned}$$

Question62

Let $f: R \rightarrow R$ and $g: R \rightarrow R$ be continuous functions. Then the value of the integral $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}[f(x)+f(-x)][g(x)-g(-x)]dx$ is MHT CET 2023 (10 May Shift 2)

Options:

A. π

B. 1

C. -1

D. 0

Answer: D

Solution:

Let $h(x)=[f(x)+f(-x)][g(x)-g(-x)]$

$$\begin{aligned}\therefore\quad h(-x) &= [f(-x)+f(x)][g(-x)-g(x)]\\ &= -[f(x)+f(-x)][g(x)-g(-x)]\\ &= -h(x)\end{aligned}$$

$\therefore\quad h(x)$ is an odd function.

$\therefore\quad \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} h(x)=0$

Question63

$\int_0^{\pi}\frac{x\tan x}{\sec x+\cos x}\,dx=$ **MHT CET 2023 (10 May Shift 2)**

Options:

A. $\frac{\pi}{8}$

B. $-\frac{\pi^2}{8}$

C. $\frac{\pi^2}{4}$

D. $-\frac{\pi^2}{4}$

Answer: C

Solution:

$$\text{Let } I = \int_0^{\pi} \frac{x \tan x}{\sec x + \cos x} dx \quad \ldots \text{(i)}$$

$$\therefore \quad I = \int_0^{\pi} \frac{(\pi - x) \tan x}{\sec x + \cos x} dx \ldots \text{(ii)}$$

$$\ldots \left[\because \int_0^a f(x) dx = \int_0^a f(a - x) dx\right]$$

Adding (i) and (ii), we get

$$2I = \pi \int_0^{\pi} \frac{\tan x}{\sec x + \cos x} dx$$

$$\Rightarrow I = \frac{\pi}{2} \int_0^{\pi} \frac{\sin x}{1 + \cos^2 x} dx$$

Put $\cos x = t \Rightarrow \sin x \, dx = -dt$

$$\therefore \quad I = -\frac{\pi}{2} \int_1^{-1} \frac{dt}{1 + t^2}$$

$$= -\frac{\pi}{2} \left[\tan^{-1} t\right]_1^{-1}$$

$$= \left(-\frac{\pi}{2}\right)\left(-\frac{\pi}{2}\right) = \frac{\pi^2}{4}$$

Question64

$\int_0^1 \cos^{-1} x dx =$ **MHT CET 2023 (10 May Shift 2)**

Options:

A. -1

B. 0

C. 1

D. 2

Answer: C

Solution:

$$I = \int_0^1 \left(\cos^{-1} x\right)(1) dx$$

$$= \left[\cos^{-1} x \cdot x - \int \left(\frac{-1}{\sqrt{1 - x^2}} \cdot x\right) dx\right]_0^1$$

$$\text{Let} \quad = \left[x \cdot \cos^{-1} x + \int \frac{x}{\sqrt{1 - x^2}} dx\right]_0^1$$

$$= \left[x \cos^{-1} x - \sqrt{1 - x^2}\right]_0^1$$

$$= \left[1 \cdot \cos^{-1}(1) - \sqrt{1 - (1)^2}\right] - \left[0 \cdot \cos^{-1}(0) - \sqrt{1 - 0^2}\right]$$

$$= 0 - (-1)$$

$$= 1$$

Question65

If $\int_0^{\frac{1}{2}} \frac{x^2}{\left(1-x^2\right)^{\frac{3}{2}}} \mathrm{~d}x=\frac{\mathrm{k}}{6}$, then the value of k is MHT CET 2023 (10 May Shift 1)

Options:

A. $2\sqrt{3}-\pi$

B. $2\sqrt{3}+\pi$

C. $3\sqrt{2}+\pi$

D. $3\sqrt{2}-\pi$

Answer: A

Solution:

Let $\mathrm{I}=\int_0^{\frac{1}{2}} \frac{x^2}{\left(1-x^2\right)^{\frac{3}{2}}} \mathrm{~d}x$

Put $x=\sin\theta$

$$\begin{aligned}
& \Rightarrow \mathrm{d}x=\cos\theta \mathrm{d}\theta \\
& \left(1-x^2\right)^{\frac{3}{2}}=\left(1-\sin^2\theta\right)^{\frac{3}{2}} \\
& =\left(\cos^2\theta\right)^{\frac{3}{2}} \\
& =\cos^3\theta \\
& \therefore \quad I=\int_0^{\frac{\pi}{6}} \frac{\sin^2\theta \cdot \cos\theta d\theta}{\cos^3\theta} \\
& =\int_0^{\frac{\pi}{6}} \tan^2\theta d\theta \\
& =\int_0^{\frac{\pi}{6}}\left(\sec^2\theta-1\right) d\theta \\
& =[\tan\theta]_0^{\frac{\pi}{6}}-[\theta]_0^{\frac{\pi}{6}} \\
& =\left(\tan\frac{\pi}{6}-\tan 0\right)-\left(\frac{\pi}{6}-0\right) \\
& =\frac{1}{\sqrt{3}}-\frac{\pi}{6} \\
& =\frac{\sqrt{3}}{3}-\frac{\pi}{6} \\
& =\frac{2\sqrt{3}-\pi}{6} \\
& \text{But, } \int_0^{\frac{1}{2}} \frac{x^2}{\left(1-x^2\right)^{\frac{3}{2}}} \mathrm{~d}x=\frac{\mathrm{k}}{6} \quad \ldots\text{[Given]} \\
& \therefore \quad \mathrm{k}=2\sqrt{3}-\pi
\end{aligned}$$

Question66

$\int_1^2 \frac{\mathrm{d}x}{\left(x^2-2x+4\right)^{\frac{3}{2}}}=\frac{\mathrm{k}}{\mathrm{k}+5}$, then k has the value MHT CET 2023 (09 May Shift 2)

Options:

A. 1

B. 2

C. -1

D. -2

Answer: A

Solution:

$$\text{Let} \quad \begin{aligned} I &= \int_1^2 \frac{dx}{(x^2-2x+4)^{\frac{3}{2}}} \\ &= \int_1^2 \frac{dx}{[(x-1)^2+3]^{\frac{3}{2}}} \end{aligned}$$

Put $x-1=\sqrt{3}\tan\theta$

$dx=\sqrt{3}\sec^2\theta d\theta$

When $x=1, \theta=0$

When $x=2, \theta=\frac{\pi}{6}$

$$\begin{aligned} \therefore \quad I &= \int_0^{\frac{\pi}{6}} \frac{\sqrt{3}\sec^2\theta}{[3\tan^2\theta+3]^{\frac{3}{2}}} d\theta \\ &= \int_0^{\frac{\pi}{6}} \frac{\sqrt{3}\sec^2\theta}{[3(1+\tan^2\theta)]^{\frac{3}{2}}} \\ &= \int_0^{\frac{\pi}{6}} \frac{\sqrt{3}\sec^2\theta}{3\cdot\sqrt{3}(\sec^2\theta)^{\frac{3}{2}}} \\ &= \int_0^{\frac{\pi}{6}} \frac{1}{3}\cdot\frac{\sec^2\theta}{\sec^3\theta} \\ &= \frac{1}{3}\int_{\infty}^{\frac{\pi}{6}} \cos\theta \\ &= \frac{1}{3}[\sin\theta]_0^{\frac{\pi}{6}} \\ I &= \frac{1}{3}\left[\sin\frac{\pi}{6}-\sin 0\right] \\ I &= \frac{1}{6} \\ \therefore \quad & \frac{k}{k+5}=\frac{1}{6} \\ 6k &= k+5 \end{aligned}$$

$\therefore k=1$

Question67

If $f(x)$ is a function satisfying $f'(x)=f(x)$ with $f(0)=1$ and $g(x)$ is a function that satisfies $f(x)+g(x)=x^2$. Then the value of the integral $\int_0^1 f(x)g(x)dx$ is MHT CET 2023 (09 May Shift 1)

Options:

A. $e-\frac{e^2}{2}-\frac{5}{2}$

B. $e+\frac{e^2}{2}-\frac{3}{2}$

C. $e-\frac{e^2}{2}-\frac{3}{2}$

D. $e+\frac{e^2}{2}+\frac{5}{2}$

Answer: C

Solution:

As $f'(x) = f(x)$

$\frac{f'(x)}{f(x)} = 1$

Integrating on both sides, we get

$\log f(x) = x + c$

As $f(0) = 1$

$\therefore \quad (i) \Rightarrow c = 0$
$\therefore \quad \log f(x) = x$
$\therefore \quad f(x) = e^x$

As $f(x) + g(x) = x^2$
$g(x) = x^2 - e^x$

$\therefore \quad f(x)g(x) = e^x \left(x^2 - e^x\right)$

$$= \int_0^1 \left(e^x x^2 - e^{2x}\right) dx$$
$$= \left[\left(x^2 - 2x + 2\right) e^x\right]_0^1 - \frac{1}{2}e^2 + \frac{1}{2}$$
$$= e - \frac{1}{2}e^2 - \frac{3}{2}$$

Question68

$\frac{\pi}{2}$ The value of $\int_0 \frac{dx}{1+\tan^3 x}$ MHT CET 2022 (11 Aug Shift 1)

Options:

A. 0

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. 1

Answer: B

Solution:

$$\int_0^{\frac{\pi}{2}} \frac{dx}{1+\tan^3 x} = \int_0^{\frac{\pi}{2}} \frac{\cos^3 x}{\cos^3 x + \sin^3 x} dx = \frac{\frac{\pi}{2} - 0}{2} = \frac{\pi}{4}$$
$$\left[\because \int_a^b \frac{f(x)dx}{f(x) + f(a+b-x)} = \frac{b-a}{2}\right]$$

Question69

The value of $\int \cos(\log_e(x))dx$ is equal to (where C is a constant of integration.) MHT CET 2022 (11 Aug Shift 1)

Options:

A. $x[\cos(\log x) - \sin(\log x)] + C$

B. $\frac{x}{2}[\sin(\log x) - \cos(\log x)] + C$

C. $\frac{x}{2}[\sin(\log x) + \cos(\log x)] + C$

D. $x[\cos(\log x) + \sin(\log x)] + C$

Answer: C

Solution:

$$\int \cos(\log_e x) \mathrm{d}x \operatorname{let}_{\log_e x = t}$$
$$\Rightarrow \mathrm{d}x = e^t$$
$$\Rightarrow I = \int \cos t \cdot e^t \,\mathrm{d}t = \cos t \cdot e^t + \sin t \cdot e^t - I \quad [\text{Integrating by parts}]$$
$$\Rightarrow 2I = e^t(\cos t + \sin t)$$
$$\Rightarrow I = \frac{x}{2}\{\cos(\log_e x) + \sin(\log_e x)\}$$

Question70

$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{e^x(x \sin x)}{e^{2x}-1} dx =$ **MHT CET 2022 (11 Aug Shift 1)**

Options:

A. 0

B. $\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. $\frac{\pi}{4}$

Answer: A

Solution:

$$\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \frac{e^x \cdot x \cdot \sin x}{e^{2x}-1} \mathrm{d}x = 0 \left[\begin{array}{c} a \\ \because \int_{-a}^{a} f(x) \mathrm{d}x = 0 \\ \text{if } f(x) \text{ is an odd function} \end{array} \right]$$

Question71

If the straight-line $x = b$ divides the area enclosed by $y = (1-x)^2_-$ $y = 0$ and $x = 0$ in two parts $R_1(0 \leq x \leq b)$ and $R_2(b \leq x \leq 1)$ such that $R_1 - R_2 = \frac{1}{4}$. Then b equals MHT CET 2022 (11 Aug Shift 1)

Options:

A. $\frac{1}{2}$

B. $\frac{1}{4}$

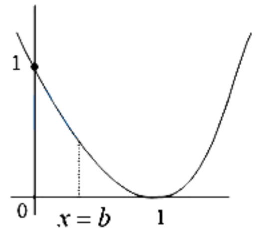
C. $\frac{3}{4}$

D. $\frac{1}{3}$

Answer: A

Solution:

$$\int_0^b (1-x)^2\,dx - \int_b^1 (1-x)^2\,dx = \frac{1}{4}$$
$$\Rightarrow \left[\frac{(1-x)^3}{-3}\right]_0^b - \left[\frac{(1-x)^3}{-3}\right]_b^1 = \frac{1}{4}$$



$$\Rightarrow \frac{(1-b)^3-(1-0)^3-(1-1)^3+(1-b)^3}{-3} = \frac{1}{4}$$
$$2(1-b)^3 - 1 = \frac{-3}{4}$$
$$\Rightarrow 2(1-b)^3 - 1 = \frac{-3}{4}$$
$$\Rightarrow (1-b)^3 = \frac{1}{8}$$
$$\Rightarrow b = \frac{1}{2}$$

Question72

$\int_{-1/2}^{1/2} \log\left(\frac{1+x}{1-x}\right) dx =$ **MHT CET 2022 (10 Aug Shift 2)**

Options:

A. 0

B. $\frac{1}{2}$

C. -1

D. $-\frac{1}{2}$

Answer: A

Solution:

$$\int_{\frac{-1}{2}}^{1/2} \log\left(\frac{1+x}{1-x}\right) dx = 0 \left[\because \int_{-a}^{a} f(x) dx = 0 \text{ if } f(x) \text{ is odd}\right]$$

Question73

$\int_0^{\pi/2} \sin^5\left(\frac{x}{2}\right) \cdot \sin x dx =$ **MHT CET 2022 (10 Aug Shift 2)**

Options:

A. $\frac{1}{7\sqrt{2}}$

B. $\frac{1}{56\sqrt{2}}$

C. $\frac{1}{14\sqrt{2}}$

D. $\frac{1}{28\sqrt{2}}$

Answer: C

Solution:

$$\int_0^{\pi/2} \sin^5 \frac{x}{2} \sin x \, \mathrm{d}x$$
$$= \int_0^{\pi/2} \sin^5 \frac{x}{2} \cdot 2 \sin \frac{x}{2} \cdot \cos \frac{x}{2} \, \mathrm{d}x$$
$$= 2 \int_0^{\pi/2} \sin^6 \frac{x}{2} \cdot \cos \frac{x}{2} \cdot \mathrm{d}x$$
$$\frac{1}{\sqrt{2}}$$
$$= 4 \int_0^6 t^6 \, \mathrm{d}t = \frac{4}{7} \left[t^7\right]_0^{\frac{1}{\sqrt{2}}} \left[\text{let } \sin \frac{x}{2} = t\right]$$

$$= \frac{4}{7} \cdot \left(\frac{1}{\sqrt{2}}\right)^7 = \frac{1}{14\sqrt{2}}$$

Question74

$\int_0^2 |2x| \mathrm{d} =$ **(where [.] denotes the greatest integer function.) MHT CET 2022 (10 Aug Shift 2)**

Options:

A. 4

B. 3

C. 2

D. 5

Answer: B

Solution:

$$\int_0^2 [2x] \mathrm{d}x = \int_0^{1/2} 0 \, \mathrm{d}x + \int_{\frac{1}{2}}^1 1 \cdot \mathrm{d}x + \int_1^{3/2} 2 \, \mathrm{d}x + \int_{\frac{3}{2}}^2 3 \, \mathrm{d}x$$
$$= 0 + \left(1 - \frac{1}{2}\right) + 2\left(\frac{3}{2} - 1\right) + 3\left(2 - \frac{3}{2}\right)$$
$$= 0 + \frac{1}{2} + 2 \times \frac{1}{2} + 3 \times \frac{1}{2} = 3$$

Question75

$\int_0^1 \frac{1}{\sqrt{3+2x-x^2}} dx =$ **MHT CET 2022 (10 Aug Shift 1)**

Options:

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{6}$

Answer: D

Solution:

$$\int_0^1 \frac{1}{\sqrt{3+2x-x^2}}\,\mathrm{d}x = \int_0^1 \frac{\mathrm{d}x}{\sqrt{(2)^2-(x-1)^2}} = \left[\sin^{-1}\left(\frac{x-1}{2}\right)\right]_0^1$$
$$= \sin^{-1}(0) - \sin^{-1}\left(\frac{-1}{2}\right) = 0 - \left(-\frac{\pi}{6}\right) = \frac{\pi}{6}$$

Question76

The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+\alpha^x}\,\mathrm{d}x, \alpha > 0$ is MHT CET 2022 (10 Aug Shift 1)

Options:

A. 2π

B. π

C. $\alpha\pi$

D. $\frac{\pi}{2}$

Answer: D

Solution:

Let $I = \int_{-\pi}^{\pi} \frac{\cos^2 x}{1+\alpha^x}$ ...... (i)

$I = \int_{-\pi}^{\pi} \frac{\cos^2(-x)}{1+\alpha^{-x}}\mathrm{d}x = \int_{-\pi}^{\pi} \frac{\alpha^x \cos^2 x}{1+\alpha^x}\mathrm{d}x$ ...... (ii)

from (i)+(ii)

$$2I = \int_{-\pi}^{\pi} \frac{(1+\alpha^x)\cos^2 x}{(1+\alpha^x)}\mathrm{d}x = \int_{-\pi}^{\pi} \cos^2 x\,\mathrm{d}x$$
$$\Rightarrow 2I = 2\int_0^{\pi} \cos^2 x\,\mathrm{d}x \quad [\because \cos^2(\pi - x) = \cos^2 x]$$
$$\Rightarrow I = \int_0^{\pi} \frac{1+\cos 2x}{2}\,\mathrm{d}x \left[\frac{x}{2} + \frac{\sin 2x}{4}\right]_0^{\pi} = \frac{\pi}{2}$$

Question77

$\int_{-3}^{0} x\sqrt{x+4}dx =$ **MHT CET 2022 (10 Aug Shift 1)**

Options:

A. $\frac{-94}{15}$

B. $\frac{94}{15}$

C. $\frac{-34}{15}$

D. $\frac{64}{15}$

Answer: A

Solution:

$\int_{-3}^{0} x\sqrt{x+4}dx$

let $x+4=t$

$$\Rightarrow x=t-4$$
$$\Rightarrow \mathrm{d}x=\mathrm{d}t$$

when $x=-3, t=1$

when $x=0, t=4$

$$\int_1^4 (t-4)\sqrt{t}\,\mathrm{d}t = \int_1^4 \left(t^{3/2}-4t^{1/2}\right)\mathrm{d}t = \left[\frac{2}{5}t^{5/2}-\frac{8}{3}t^{3/2}\right]_1^4$$
$$= \frac{2}{5}(32-1)-\frac{8}{3}(8-1) = \frac{186-280}{15} = -\frac{94}{15}$$

Question78

$\int_0^1 |5x-3|\mathrm{d}x =$ **MHT CET 2022 (08 Aug Shift 2)**

Options:

A. $\frac{23}{10}$

B. $\frac{13}{10}$

C. $\frac{31}{10}$

D. $\frac{1}{2}$

Answer: B

Solution:

$$\int_0^1 |5x-3|\mathrm{d}x = \int_0^{3/5}(3-5x)\mathrm{d}x + \int_{3/5}^1 (5x-3)\mathrm{d}x$$
$$= \left[3x-\frac{5x^2}{2}\right]_0^{3/5} + \left[\frac{5x^2}{2}-3x\right]_{3/5}^1$$
$$= \left(\frac{9}{5}-\frac{9}{10}\right)-0+\left(\frac{5}{2}-3\right)-\left(\frac{9}{10}-\frac{9}{5}\right)$$
$$= \frac{13}{10}$$

Question79

$\int_0^{\pi/4} \sqrt{1-\sin 2x} dx=$ **MHT CET 2022 (08 Aug Shift 2)**

Options:

A. $\sqrt{2}+1$

B. $1+2\sqrt{2}$

C. $\sqrt{2}-1$

D. $2\sqrt{2}-1$

Answer: C

Solution:

$$\int_0^{\pi/4} \sqrt{1-\sin 2x} \, \mathrm{d}x=\int_0^{\pi/4} \sqrt{\cos^2 x+\sin^2 x-2 \sin x \cdot \cos x} \, \mathrm{d}x$$
$$=\int_0^{\pi/4} \sqrt{(\cos x-\sin x)^2} \, \mathrm{d}x=\int_0^{\pi/4} |\cos x-\sin x| \mathrm{d}x$$
$$=\int_0^{\pi/4}(\cos x-\sin x) \mathrm{d}x=[\sin x+\cos x]_0^{\pi/4}$$
$$=\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)-(0+1)=-1$$

Question80

If $g(x)=\int_0^x \cos^4 t \, \mathrm{d}t$, then $g(x+\pi)$ equals MHT CET 2022 (08 Aug Shift 2)

Options:

A. $g(x)+g(\pi)$

B. $g(x)-g(\pi)$

C. $\frac{g(x)}{g(\pi)}$

D. $g(x) \cdot g(\pi)$

Answer: A

Solution:

$$g(x)=\int_0^x \cos^4 t \, \mathrm{d}t$$
$$\Rightarrow g(x+\pi)=\int_0^{x+\pi} \cos^4 t \, \mathrm{d}t=\int_0^x \cos^4 t \, \mathrm{d}t+\int_x^{x+\pi} \cos^4 t \, \mathrm{d}t$$
$$=g(x)+\int_x^{x+\pi} \cos^4 t \, \mathrm{d}t$$
$$=g(x)+\int_0^\pi \cos^4 t \, \mathrm{d}t$$

[as $\cos^4 t$ is a periodic function with period π] $=g(x)+g(\pi)$

Question81

$\int_0^{\frac{\pi}{4}} \sec^4 x dx =$ **MHT CET 2022 (08 Aug Shift 1)**

Options:

A. $\frac{2}{3}$

B. $\frac{1}{3}$

C. $\frac{4}{3}$

D. 1

Answer: C

Solution:

$$\int_0^{\frac{\pi}{4}} \sec^4 x \, \mathrm{d}x = \int_0^{\frac{\pi}{4}} \left(\tan^2 x + 1\right) \sec^2 x \, \mathrm{d}x$$
$$= \left[\frac{\tan^3 x}{3} + \tan x\right]_0^{\frac{\pi}{4}}$$
$$= \frac{1}{3} + 1$$
$$= \frac{4}{3}$$

Question82

$\int_{-1}^1 \log\left(\frac{2-x}{2+x}\right) dx =$ **MHT CET 2022 (08 Aug Shift 1)**

Options:

A. 1

B. -1

C. 0

D. 2

Answer: C

Solution:

$$\int_{-1}^1 \log\left(\frac{2-x}{2+x}\right) \mathrm{d}x = 0$$
$$\because \log\left(\frac{2-x}{2+x}\right) \text{ is an odd function.}$$

Question83

$\int_0^1 \sqrt{\frac{1-x}{1+x}} dx =$ **MHT CET 2022 (07 Aug Shift 2)**

Options:

A. $\frac{\pi}{4}+1$

B. $\frac{\pi}{2}+1$

C. $\frac{\pi}{4}-1$

D. $\frac{\pi}{2}-1$

Answer: D

Solution:

$$\int_0^1 \sqrt{\frac{1-x}{1+x}} dx=\int_0^1 \frac{1-x}{\sqrt{1-x^2}} dx=\int_0^1 \frac{1}{\sqrt{1-x^2}} dx+\frac{1}{2} \int_0^1 \frac{-2 x}{\sqrt{1-x^2}} dx$$
$$=\left[\sin ^{-1} x\right]_0^1+\left[\sqrt{1-x^2}\right]_0^1$$
$$=\sin ^{-1}(1)-\sin ^{-1}(0)+\sqrt{1-1^2}-\sqrt{1-0^2}$$
$$=\frac{\pi}{2}-0+0-1$$
$$=\frac{\pi}{2}-1$$

Question84

$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} f(x) dx=$**Where** $f(x)=\sin |x|+\cos |x|, x \in\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. **MHT CET 2022 (07 Aug Shift 1)**

Options:

A. 0

B. 8

C. 4

D. 2

Answer: C

Solution:

$$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} f(x) dx=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}(\sin |x|+\cos |x|) dx$$
$$=2 \int_0^{\frac{\pi}{2}}(\sin |x|+\cos |x|) dx \quad [\because \text { even function }]$$
$$\frac{\pi}{2}$$
$$=2 \int_0(\sin x+\cos x) dx \quad[\because x>0]$$
$$=2[-\cos x+\sin x]_0^{\frac{\pi}{2}}=4$$

Question85

The value of the integral $\int_0^1 \sqrt{\frac{1-x}{1+x}}dx$ **is MHT CET 2022 (06 Aug Shift 2)**

Options:

A. $\left(\frac{\pi}{2}\right)-1$

B. -1

C. $\left(\frac{\pi}{2}\right)+1$

D. 1

Answer: A

Solution:

$$\int_0^1 \sqrt{\frac{1-x}{1+x}}dx=\int_0^1 \frac{1-x}{\sqrt{1-x^2}}dx=\int_0^1 \frac{dx}{\sqrt{1-x^2}}+\int_0^1 \frac{-2x}{2\sqrt{1-x^2}}dx$$
$$=\left[\sin^{-1}x\right]_0^1+\left[\sqrt{1-x^2}\right]_0^1$$
$$=\left\{\sin^{-1}(1)-\sin^{-1}(0)\right\}+\left\{\sqrt{1-1^2}-\sqrt{1-0^2}\right\}$$
$$=\frac{\pi}{2}-0+0-1$$
$$=\frac{\pi}{2}-1$$

Question86

$\int_0^2 [x]dx+\int_0^2 |x-1|dx=$**(where** $[x]$ **denotes the greatest integer function.) MHT CET 2022 (06 Aug Shift 2)**

Options:

A. 3

B. 4

C. 1

D. 2

Answer: D

Solution:

$$\int_0^2 [x]dx+\int_0^2 [x-1]dx$$
$$=\int_0^1 0.\,dx+\int_1^2 1.\,dx+\int_0^1 (1-x)dx+\int_1^2 (x-1)dx$$
$$=[0]_0^1+[x]_1^2+\left[x-\frac{x^2}{2}\right]_0^1+\left[\frac{x^2}{2}-x\right]_1^2$$
$$=0+1+\frac{1}{2}+\frac{1}{2}=2$$

Question87

$\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ **is equal to MHT CET 2022 (06 Aug Shift 2)**

Options:

A. -2

B. $-2-2\sqrt{2}$

C. 2

D. $-2\sqrt{2}$

Answer: C

Solution:

$$\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x} = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1}{2}\sec^2 \frac{x}{2} dx = \left[\tan \frac{x}{2}\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$= \left(\tan \frac{3\pi}{8} - \tan \frac{\pi}{8}\right) = \left(\cot \frac{\pi}{8} - \tan \frac{\pi}{8}\right) = \frac{\cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8}}{\sin \frac{\pi}{8} \cdot \cos \frac{\pi}{8}}$$

$$= \frac{2\cos \frac{\pi}{4}}{\sin \frac{\pi}{4}} = 2$$

Question88

$\frac{\pi}{2}$ **If** $\int_0 \frac{\cot x}{\cot x + \operatorname{cosec} x} dx = m(\pi + n)$**, then (m.n) equals MHT CET 2022 (06 Aug Shift 1)**

Options:

A. $\frac{1}{2}$

B. -1

C. $-\frac{1}{2}$

D. 1

Answer: B

Solution:

$$\int_0^{\frac{\pi}{2}} \frac{\cot x}{\cot x+\operatorname{cosec} x} d x=\int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x+1} d x$$
$$=\int_0^{\frac{\pi}{2}} \frac{2 \cos ^2 \frac{x}{2}-1}{2 \cos ^2 \frac{x}{2}} d x=\int_0^{\frac{\pi}{2}}\left(1-\frac{1}{2} \sec ^2 \frac{x}{2}\right) d x$$
$$=\left[x-\tan \frac{x}{2}\right]_0^{\pi / 2}=\frac{\pi}{2}-1=\frac{1}{2}(\pi+(-2))$$
$$\Rightarrow m=\frac{1}{2} \text { and } n=-2$$
$$\Rightarrow \mathrm{m} \times \mathrm{n}=\frac{1}{2} \times(-2)=-1$$

Question89

$\int_{\log \frac{1}{2}}^{\log 2} \sin \left(\frac{e^x-1}{e^x+1}\right) dx=$ **MHT CET 2022 (06 Aug Shift 1)**

Options:

A. $2 \log 2$

B. $-2 \log 2$

C. $\frac{1}{2}$

D. 0

Answer: D

Solution:

$$\int_{\log \frac{1}{2}}^{\log 2} \sin \left(\frac{e^x-1}{e^x+1}\right) dx=\int_{-\log 2}^{\log 2} \sin \left(\frac{e^x-1}{e^x+1}\right) dx=0$$
$$\left[\because \sin \left(\frac{e^x-1}{e^x+1}\right) \text { is an odd function }\right]$$

Question90

If $\int_0^k \frac{d x}{2+8 x^2}=\frac{\pi}{16}$, then value of k is **MHT CET 2022 (06 Aug Shift 1)**

Options:

A. 4

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. 2

Answer: B

Solution:

$$\int_0^k \frac{dx}{2+8x^2} = \frac{\pi}{16} \Rightarrow \frac{1}{2}\int_0^k \frac{dx}{1+(2x)^2} = \frac{\pi}{16}$$

$$\Rightarrow \frac{1}{2}\left[\frac{\tan^{-1}(2x)}{2}\right]_0^k = \frac{\pi}{16}$$

$$\Rightarrow \tan^{-1}(2k) = \frac{\pi}{16}$$

$$\Rightarrow 2k = 1$$

$$\Rightarrow k = \frac{1}{2}$$

Question91

$\int_0^{2\pi}(\sin x + |\sin x|)dx =$ **MHT CET 2022 (05 Aug Shift 2)**

Options:

A. 0

B. 8

C. 4

D. 1

Answer: C

Solution:

$$\int_0^{2\pi}(\sin x + |\sin x|)dx =$$

$$= \int_0^{\pi}(\sin x + \sin x)dx + \int_0^{\pi}(\sin x - \sin x)dx$$

$$= \int_0^{\pi} 2\sin x dx + 0$$

$$= [-2\cos x]_0^{\pi} = 4$$

Question92

$\int_0^1 x(1-x)^n dx =$ **MHT CET 2022 (05 Aug Shift 2)**

Options:

A. $\frac{n+3}{(n+1)(n+2)}$

B. $\frac{1}{(n+1)(n+2)}$

C. $\frac{2n+3}{(n+1)(n+2)}$

D. $\frac{4}{(n+1)(n+2)}$

Answer: B

Solution:

$$\int_0^1 x(1-x)^n dx=\int_0^1 (1-x)^n dx\left[\because \int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$$
$$=\int_0^1\left(x^n-x^{n+1}\right)dx=\left[\frac{x^{n+1}}{n+1}-\frac{x^{n+2}}{n+2}\right]_0^1$$
$$=\frac{1}{n+1}-\frac{1}{n+2}=\frac{1}{(n+1)(n+2)}$$

Question93

Let $f:[-1,2]\rightarrow[0,\infty)$ be a continuous function such that $f(x)=f(1-x),\forall x\in[-1,2]$. If $R_1=\int_{-1}^{2} xf(x)dx$ and R_2 is the area of the region bounded by $y=f(x),x=-1,x=2$ and the X-asis. Then MHT CET 2022 (05 Aug Shift 1)

Options:

A. $2R_1=R_2$

B. $R_1=3R_2$

C. $R_1=2R_2$

D. $3R_1=R_2$

Answer: A

Solution:

Here, $R_1=\int_{-1}^{2} xf(x)dx$ ....(i)

$\Rightarrow R_1=\int_{-1}^{2}(1-x)f(1-x)dx=\int_{-1}^{2}(1-x)f(x)$ ....(ii)

[as $\int_a^b f(x)dx=\int_a^b f(a+b-x)$ also $f(1-x)=f(x)$].

from (i) + (ii) $2R_1=\int_{-1}^{2} f(x)dx$ ....(iii)

and $R_2=\int_{-1}^{2} f(x)dx$ ....(iv)

from (iii) and (iv) $2R_1=R_2$

Question94

$\int_0^{\frac{\pi}{2}}\frac{\sin x}{1+\cos^2 x}dx$ **has the value MHT CET 2022 (05 Aug Shift 1)**

Options:

A. $-\frac{\pi}{4}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. 0

Answer: B

Solution:

$$\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 \mathrm{x}} \mathrm{dx} = -\int_1^0 \frac{\mathrm{dt}}{1+\mathrm{t}^2} [\text{ let } \cos \mathrm{x} = \mathrm{t}]$$
$$= -\left[\tan^{-1} \mathrm{t}\right]_1^0$$
$$= -\left(0 - \frac{\pi}{4}\right)$$
$$= \frac{\pi}{4}$$

Question95

Let $[t]$ denot the greatest integer less than or equal to t. Then the value of $\int_1^2 |2x - [3x]| dx$ is MHT CET 2022 (05 Aug Shift 1)

Options:

A. 1

B. $\frac{3}{2}$

C. 2

D. 0

Answer: A

Solution:

$$\int_1^2 |2x - [3x]| dx = \int_1^{\frac{4}{3}} |2x - 3| dx + \int_{\frac{4}{3}}^{\frac{5}{3}} |2x - 4| dx + \int_{\frac{5}{3}}^2 |2x - 5| dx$$
$$= \int_1^{\frac{4}{3}} (3 - 2x) dx + \int_{\frac{4}{3}}^{\frac{5}{3}} (4 - 2x) dx + \int_{\frac{5}{3}}^2 (5 - 2x) dx$$
$$= \left[3x - x^2\right]_1^{4/3} + \left[4x - x^2\right]_{4/3}^{5/3} + \left[5x - x^2\right]_{5/3}^2$$
$$= 3[x]_1^{4/3} + [4x]_{4/3}^{5/3} + 5\left[x^2\right]_{5/3}^2 - \left[x^2\right]_1^2$$
$$= (3 + 4 + 5) \times \frac{1}{3} - \left(2^2 - 1^2\right)$$
$$= 4 - 3 = 1$$

Question96

If $\int_a^b x^3 dx = 0$ and if $\int_a^b x^2 dx = \frac{2}{3}$, then a and b are respectively MHT CET 2022 (05 Aug Shift 1)

Options:

A. $1, -1$

B. $-1, -1$

C. 1,1

D. -1,1

Answer: D

Solution:

$$\int_a^b x^3 dx = 0 \text{ and } \int_a^b x^2 dx = \frac{2}{3}$$
$$\Rightarrow \left[\frac{x^4}{4}\right]_a^b = 0 \text{ and } \left[\frac{x^3}{3}\right]_a^b = \frac{2}{3}$$
$$\Rightarrow \frac{b^4}{4} - \frac{a^4}{4} = 0 \text{ and } \frac{b^3}{3} - \frac{a^3}{3} = \frac{2}{3}$$
$$\Rightarrow a^4 = b^4 \text{ and } b^3 - a^3 = 2$$

which is satisfied by $a = -1$ and $b = 1$

Question97

$\int_2^5 2[x] d18x = \{$ **where** $[x]$ **denotes the greatest integer function** $\leq x\}$ **MHT CET 2021 (24 Sep Shift 2)**

Options:

A. 18

B. 16

C. 12

D. 24

Answer: A

Solution:

$$\text{Let } I = \int_2^5 2[x] dx$$
$$= \int_2^3 2(2) dx + \int_3^4 2(3) dx + \int_4^5 2(4) dx$$
$$= 4[x]_2^3 + 6[x]_3^4 + 8[x]_4^5 = 4 + 6 + 8 = 18$$

Question98

$\int_0^\pi x \sin x \cos^4 x dx =$ **MHT CET 2021 (24 Sep Shift 2)**

Options:

A. $\frac{\pi}{10}$

B. $\frac{2\pi}{5}$

C. $\frac{\pi}{5}$

D. $\frac{\pi}{8}$

Answer: C

Solution:

Let $I = \int_0^{\pi} x \sin x \cos^4 x \, dx$ ...(1)

$= \int_0^{\pi} (\pi - x) \sin(\pi - x)[\cos(\pi - x)]^4 dx$

$= \int_4^{\pi} (\pi - x) \sin x [\cos x]^4 dx$ ... (2)

Eq. (1) $+(2)$ gives,

$2I = \int_0^{\pi} \pi \sin x \cos^4 x dx$

Eq. (1) $+(2)$ gives,

$2I = \int_0^{\pi} \pi \sin x \cos^4 x dx$

Put $\cos x = t \Rightarrow -\sin x dx =$

When $\mathrm{x} = 0, \mathrm{t} =$ and when $\mathrm{x} = \pi, \mathrm{t} = -1$

$2\mathrm{I} = \pi \int_1^{-1} (\mathrm{t})^4 (-\mathrm{dt})$

... (t^4 is an even function)

$\therefore \quad 2I = \frac{2\pi}{5} \left[\mathrm{t}^5\right]_0^1 \Rightarrow \mathrm{I} = \frac{\pi}{5}$

Question99

$\int_0^{\pi} \frac{1}{4+3\cos x} dx =$ **MHT CET 2021 (24 Sep Shift 1)**

Options:

A. 1

B. $\frac{\pi}{\sqrt{7}}$

C. 0

D. $\frac{2}{\sqrt{7}}$

Answer: B

Solution:

Let $\int_0^\pi \frac{1}{4+3\cos x} dx$

Put $\frac{x}{2}=t \Rightarrow \cos x=\frac{1-t^2}{1+t^2}$ and $\sec^2\frac{x}{2}\left(\frac{1}{2}\right)dx=dt \Rightarrow dt=\frac{2}{1+t^2}dt$

When $\mathrm{x}=0, \mathrm{t}=0$ and when $\mathrm{x}=\pi, \mathrm{t}=\infty$

$$\begin{aligned}
\therefore \quad \mathrm{I} &= \int_0^\infty \frac{1}{4+3\left(\frac{1-t^2}{1+t^2}\right)} \times \frac{2}{1+t^2} \\
&= \int_0^\infty \frac{(1+t^2)}{4(1+t^2)+3(1-t^2)} \times \frac{2}{1+t^2}dt = \int_0^\infty \frac{2}{7+t^2}dt = \frac{2}{7}\int_0^\infty \frac{dt}{1+\left(\frac{1}{\sqrt{7}}\right)^2} \\
&= \frac{2}{7}\left[\tan^{-1}\left(\frac{t}{7}\right)\right]_0^\infty \frac{1}{\left(\frac{1}{\sqrt{7}}\right)} = \frac{2}{\sqrt{7}}\left[\tan^{-1}\infty - \tan^{-1}0\right] = \frac{2}{\sqrt{7}} \times \frac{\pi}{2} = \frac{\pi}{\sqrt{7}}
\end{aligned}$$

Question100

$\int_0^1 |5x-3| dx =$ **MHT CET 2021 (23 Sep Shift 2)**

Options:

A. $\frac{13}{10}$

B. 1

C. $\frac{3}{10}$

D. $\frac{1}{2}$

Answer: A

Solution:

Let $\mathrm{I}=\int_0^1 |5\mathrm{x}-3| d\mathrm{x}$

$$\begin{aligned}
&5x-3=0 \Rightarrow x=\frac{3}{5} \\
&\therefore \quad I=\int_0^{\frac{3}{5}} -(5x-3)dx + \int_{\frac{3}{5}}^1 (5x-3)dx \\
&= \frac{-5}{2}\left[x^2\right]_0^{\frac{3}{5}} + 3[x]_0^{\frac{3}{5}} + \frac{5}{2}\left[x^2\right]_{\frac{3}{5}}^1 - 3[x]_{\frac{3}{5}}^1 \\
&= \left(\frac{-5}{2}\right)\left(\frac{9}{25}\right) + 3\left(\frac{3}{5}\right) + \frac{5}{2}\left(1-\frac{9}{25}\right) - 3\left(1-\frac{3}{5}\right) \\
&= \frac{-45}{50} + \frac{9}{5} + \left(\frac{5}{2}\right)\left(\frac{16}{25}\right) - 3\left(\frac{2}{5}\right) = \frac{-45}{50} + \frac{3}{5} + \frac{8}{5} \\
&= \frac{-45+110}{50} = \frac{65}{50} = \frac{13}{10}
\end{aligned}$$

Question101

$\int_0^4 x[x]dx =$(where $[x]$ denotes greatest integer function not greater than x) **MHT CET 2021 (23 Sep Shift 1)**

Options:

A. 17

B. 24

C. $\frac{21}{2}$

D. $\frac{33}{2}$

Answer: A

Solution:

Let $I=\int_0^4 x[x]dx$

$$\begin{aligned}
\therefore I &= \int_0^1 (0)dx + \int_1^2 xdx + \int_2^3 2xdx + \int_3^4 3xdx \\
&= \left[\frac{x^2}{2}\right]_1^2 + \left[\frac{2x^2}{2}\right]_2^3 + \left[\frac{3x^2}{2}\right]_3^4 \\
&= \frac{1}{2}(4-1)+(9-4)+\left(\frac{3}{2}\right)(16-9) = \frac{3}{2}+5+\frac{21}{2} = 17
\end{aligned}$$

Question102

$\int_0^{\pi/2} \log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx =$ **MHT CET 2021 (23 Sep Shift 1)**

Options:

A. 0

B. $4\log 3$

C. $\frac{1}{2}$

D. $2\log 4$

Answer: A

Solution:

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx \quad \ldots (1)$$

$$= \int_0^{\frac{\pi}{2}} \log\left[\frac{4+3\sin\left(\frac{\pi}{2}-x\right)}{4+3\cos\left(\frac{\pi}{2}-x\right)}\right]dx = \int_0^{\frac{\pi}{2}} \log\left[\frac{4+3\cos x}{4+3\sin x}\right]dx \quad \ldots (2)$$

Eq. (1) $+(2)$ gives,

$$2I = \log\left[\frac{4+3\sin x}{4+3\cos x} \times \frac{4+3\cos x}{4+3\sin x}\right]dx = \int_0^{\frac{\pi}{2}} (\log 1)dx = 0$$

Question103

A fair coin is tossed 4 times. If X a random variable which indicates number of heads, then $\mathrm{P}[\mathrm{X} < 3] =$ MHT CET 2021 (22 Sep Shift 2)

Options:

A. $\frac{10}{16}$

B. $\frac{1}{16}$

C. $\frac{12}{16}$

D. $\frac{11}{16}$

Answer: D

Solution:

A coin is tossed 4 times

$$\therefore \mathrm{n(S)} = 2^4 = 16$$

Following possibilities exist.

(i) All Heads $\Rightarrow 1$ way

(ii) 3 Heads, 1 Tail $\Rightarrow \frac{4!}{3!} = 4$ ways

(iii) 2 Heads, 2 Tails $\Rightarrow \frac{4!}{2!2!} = 6$ ways

(iv) 1 Head, 3 Tails $\Rightarrow \frac{4!}{3!} = 4$ ways

(v) 0 head, 4 Tails $\Rightarrow 1$ way

$\therefore$ Required probability.

$$= \mathrm{P}(\mathrm{x} = 0,1,2) = \frac{1}{16} + \frac{4}{16} + \frac{6}{16}$$
$$= \frac{11}{16}$$

Question104

$\int_0^2 |2x-3|dx =$ **MHT CET 2021 (22 Sep Shift 2)**

Options:

A. $\frac{3}{10}$

B. $\frac{5}{2}$

C. $\frac{10}{3}$

D. $\frac{2}{5}$

Answer: B

Solution:

LetWhen $x = \frac{3}{2}, 2x - 3 = 0$

$$\therefore \mathrm{I} = \int_0^{3/2} (3 - 2\mathrm{x})\mathrm{dx} + \int_{\frac{3}{2}}^{2} (2\mathrm{x} - 3)\mathrm{dx} = [3\mathrm{x}]_0^{3/2} - \frac{2}{2}\left[\mathrm{x}^2\right]_{3/2}^{2} - [3\mathrm{x}]_{3/2}^{2}$$
$$= \left(\frac{9}{2}\right) - \left(\frac{9}{4}\right) + \left(4 - \frac{9}{4}\right) - 3\left(2 - \frac{3}{2}\right) = \frac{5}{2}$$

Question105

$\int_0^a \sqrt{\frac{a-x}{x}} dx = \frac{k}{2}$, **then** $k =$ **MHT CET 2021 (22 Sep Shift 2)**

Options:

A. $\pi \mathrm{a}$

B. $\frac{\pi \mathrm{a}}{2}$

C. $\frac{5\pi \mathrm{a}}{2}$

D. $\frac{3\pi \mathrm{a}}{2}$

Answer: A

Solution:

We have $\int_0^3 \sqrt{\frac{a-x}{x}} dx = \frac{k}{2}$

Put $x = a\sin^2\theta \Rightarrow dx = a(2\sin\theta)\cos\theta d\theta = 2a\sin\theta\cos\theta d\theta$

When $\mathrm{x} = 0, \theta = 0$ and when $\mathrm{x} = \mathrm{a}, \theta = \frac{\pi}{2}$

$$\therefore \int_0^{\frac{\pi}{2}} \sqrt{\frac{\mathrm{a} - \mathrm{a}\sin^2\theta}{\mathrm{a}\sin^2\theta}} (2\mathrm{a}\sin\theta\cos\theta)\mathrm{d}\theta = \frac{\mathrm{k}}{2}$$
$$\int_0^{\frac{\pi}{2}} \sqrt{\frac{\cos^2\theta}{\sin^2\theta}} 2(\mathrm{a}\sin\theta\cos\theta)\mathrm{d}\theta = \frac{\mathrm{k}}{2} \Rightarrow \int_0^{\frac{\pi}{2}} \sqrt{\frac{\cos\theta}{\sin\theta}} 2(\mathrm{a}\sin\theta\cos\theta)\mathrm{d}\theta = \frac{\mathrm{k}}{2}$$
$$\int_0^{\frac{\pi}{2}} 2\mathrm{a}\cos\theta^2 \, \mathrm{d}\theta = \frac{\mathrm{k}}{2} \Rightarrow 2\mathrm{a}\int_0^{\frac{\pi}{2}} \frac{1 + \cos 2\theta}{2} \, \mathrm{d}\theta = \frac{\mathrm{k}}{2}$$
$$\int_0^{\frac{\pi}{2}} 2\mathrm{a}\cos\theta^2 \, \mathrm{d}\theta = \frac{\mathrm{k}}{2} \Rightarrow 2\mathrm{a}\int_0^{\frac{\pi}{2}} \frac{1 + \cos 2\theta}{2} \, \mathrm{d}\theta = \frac{\mathrm{k}}{2}$$
$$\mathrm{a}\left[\left(\frac{\pi}{2} + 0\right) - (0 + 0)\right] = \frac{\mathrm{k}}{2} \Rightarrow \mathrm{a}\frac{\pi}{2} = \frac{\mathrm{k}}{2} \Rightarrow \mathrm{k} = \pi \mathrm{a}$$

Question106

The value of $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$ is MHT CET 2021 (22 Sep Shift 1)

Options:

A. 2

B. -1

C. 1

D. 0

Answer: D

Solution:

$$\text{Let } I=\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$$

$$=\int_0^1 \tan^{-1}\left[\frac{x+(x-1)}{1+x(1-x)}\right]dx=\int_0^1 \tan^{-1}\left[\frac{x+(x-1)}{1-(x-1)(x)}\right]dx$$

$$=\int_0^1 \left[\tan^{-1}x+\tan^{-1}(x-1)dx=\int_0^1 \tan^{-1}x dx+\int_0^1 \tan^{-1}(x-1)dx\right.$$

$$=\left[\left[x\tan^{-1}x\right]_0^1-\frac{1}{2}\int_0^1 \frac{2x}{1+x^2}dx\right]+\left[x\tan^{-1}(x-1)\right]_0^1-\frac{1}{2}\int_0^1 \frac{2x-2+2}{1+(x-1)^2}dx\right]$$

$$=\left(\frac{\pi}{4}\right)-\frac{1}{2}\left[\log\left|1+x^2\right|\right]_0^1+0-\frac{1}{2}\int_0^1 \frac{2x-2}{1+(x-1)^2}dx-\int_0^1 \frac{dx}{1+(x-1)^2}$$

$$=\left(\frac{\pi}{4}\right)-\frac{1}{2}\log 2-\frac{1}{2}\left[\log\left|1+(x-1)^2\right|\right]_0^1-\left[\tan^{-1}(x-1)\right]_0^1$$

$$=\frac{\pi}{4}-\frac{1}{2}\log 2-\frac{1}{2}(0-\log 2)-\left[0-\tan^{-1}(-1)\right]$$

$$=\frac{\pi}{4}-\frac{1}{2}\log 2+\frac{1}{2}\log 2-\frac{\pi}{4}=0$$

Question107

$\int_{-\pi}^{\pi} \frac{x\sin x}{1+\cos^2 x}dx=$ MHT CET 2021 (22 Sep Shift 1)

Options:

A. $\frac{\pi^2}{2}$

B. π^2

C. $\frac{\pi^2}{4}$

D. 3π

Answer: A

Solution:

Let $f(x)=\frac{x \sin x}{1+\cos^2 x}$

$f(-x)=\frac{(-x)\sin(-x)}{1+\cos^2 x}=\frac{x \sin x}{1+\cos^2 x}$

$\therefore \mathrm{f(x)=f(-x) \Rightarrow f(x)}$ is an even function

Let $I=\int_{-\pi}^{\pi}\frac{x \sin x}{1+\cos^2 x}dx$

$\therefore \mathrm{I}=2\int_0^{\pi}\frac{\mathrm{x}\sin \mathrm{x}}{1+\cos^2 \mathrm{x}}\mathrm{dx}$

$=2\int_0^{\pi}\frac{(\pi-x)\sin(\pi-x)}{1+[\cos(\pi-x)]^2}dx=2\int_0^{\pi}\frac{(\pi-x)\sin x}{1+\cos^2 x}dx$

Eq. (1) $+(2)$ gives,

$$2I=2\pi\int_0^{\pi}\frac{\sin x}{1+\cos^2 \mathrm{x}}$$

Put $\cos x=t \Rightarrow \sin x dx=-dt$. Also when $x=0, t=1$ and when

$$\begin{aligned}
&\mathrm{x}=\pi, \mathrm{t}=-1\\
&\therefore 2\mathrm{I}=2\pi\int_1^{-1}\frac{-\mathrm{dt}}{1+\mathrm{t}^2}\\
&=2\pi\int_{-1}^{1}\frac{\mathrm{dt}}{1+\mathrm{t}^2}=4\pi\int_0^1\frac{\mathrm{dt}}{1+\mathrm{t}^2}=4\pi\left[\tan^{-1}\right]_0^2=4\pi\left(\frac{\pi}{4}\right)=\pi^2\\
&\therefore \mathrm{I}=\frac{\pi^2}{2}
\end{aligned}$$

Question108

$\int_5^{10}\frac{dx}{(x-1)(x-2)}=$ **MHT CET 2021 (21 Sep Shift 2)**

Options:

A. $\log\left|\frac{27}{32}\right|$

B. $\log\left|\frac{3}{4}\right|$

C. $\log\left|\frac{8}{9}\right|$

D. $\log\left|\frac{32}{27}\right|$

Answer: D

Solution:

$$\begin{aligned}
&I=\int_5^{10}\frac{dx}{(x-1)(x-2)}\\
&=\int_5^{10}\left[\frac{1}{x-1}-\frac{1}{x-2}\right](-1)dx=-\int_5^{10}\left[\frac{1}{x-1}-\frac{1}{x-2}\right]dx\\
&=-[\log|x-1|]_5^{10}+[\log|x-2|]_5^{10}=-[\log|9|-\log|4|]+[\log|8|-\log|3|]\\
&=\left[\log\left|\frac{8}{3}\right|\right]-\left[\log\left|\frac{9}{4}\right|\right]=\log\left|\frac{8}{3}\times\frac{9}{4}\right|=\log\left|\frac{32}{27}\right|
\end{aligned}$$

Question109

$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{1+e^x} dx =$ **MHT CET 2021 (21 Sep Shift 2)**

Options:

A. 1

B. 2

C. -1

D. 0

Answer: A

Solution:

$$\text{Let } I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{1+e^x} dx \qquad \ldots (1)$$

$$= \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\cos\left(\frac{\pi}{2}-\frac{\pi}{2}-x\right)}{1+e^{\left(\frac{\pi}{2}-\frac{\pi}{2}-x\right)}} dx = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\cos(-x)}{1+e^{-x}} dx = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{1+\left(\frac{1}{e^x}\right)} dx$$

$$= \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{e^x \cos x}{1+e^x} dx \qquad \ldots (2)$$

Eq. (1) + (2) gives

$$2I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x(1+e^x)}{(1+e^x)} dx = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \cos x \, dx \quad 2\int_0^{\frac{\pi}{2}} \cos x \, dx$$

$$\therefore \ I = \int_0^{\frac{\pi}{2}} \cos x \, dx = [\sin x]_0^{\frac{\pi}{2}} = 1$$

Question110

$$\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 - \sin x \cos x} dx =$$

MHT CET 2021 (21 Sep Shift 1)

Options:

A. $\frac{\pi}{4}$

B. $\frac{2}{\pi}$

C. 0

D. $\frac{\pi}{2}$

Answer: C

Solution:

Let $I=\int_0^{\pi/2}\frac{\sin x-\cos x}{1-\sin x\cos x}dx$

$$\begin{aligned}\therefore I&=\int_0^{\frac{\pi}{2}}\frac{\sin\left(\frac{\pi}{2}-x\right)-\cos\left(\frac{\pi}{2}-x\right)}{1-\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)}dx\\&=\int_0^{\pi/2}\frac{\cos x-\sin x}{1-\cos x\sin x}dx\end{aligned}$$

Eq. (1) +(2) gives

$$2\mathrm{I}=\int_0^{\frac{\pi}{2}}0\mathrm{dx}\Rightarrow\mathrm{I}=0$$

Question111

If $\mathrm{f(x)}=|\mathrm{x}-1|+|\mathrm{x}-2|+|\mathrm{x}-3|,\forall\mathrm{x}\in[1,4]$**, then** $\int_1^4\mathrm{f(x)dx}=$ **MHT CET 2021 (21 Sep Shift 1)**

Options:

A. $\frac{1}{2}$

B. 7

C. $\frac{9}{2}$

D. $\frac{19}{2}$

Answer: D

Solution:

$$\begin{aligned}&\int_1^4 f(x)dx=\int_1^4[|x-1|+|x-2|+|x-3|dx\\&=\int_1^2(x-1)+(2-x)+(3-x)dx+\int_2^3[(x-1)+(x-2)+(3-x)]dx\\&+\int_3^4[(x-1)+(x-2)+(x-3)]dx\\&=\int_1^2(4-x)dx+\int_2^3 xdx+\int_3^4(3x-6)dx\end{aligned}$$

$$= \left[4x - \frac{x^2}{2}\right]_1^2 + \left[\frac{x^2}{2}\right]_2^3 + \left[\frac{3x^2}{2} - 6x\right]_3^4$$

$$= \left[(8-2) - \left(4 - \frac{1}{2}\right)\right] + \left[\left(\frac{9}{2} - 2\right)\right] + \left[(24-24) - \left(\frac{27}{2} - 18\right)\right]$$

$$= \left(2 + \frac{1}{2}\right) + \left(\frac{5}{2}\right) + \left(\frac{9}{2}\right) = \frac{19}{2}$$

Question112

$\int_2^e \left[\frac{1}{\log x} - \frac{1}{(\log x)^2}\right] dx = a + \frac{b}{\log 2}$, **then MHT CET 2021 (20 Sep Shift 2)**

Options:

A. $a = -e, b = 2$

B. $a = e, b = -2$

C. $a = e, b = 2$

D. $a = -e, b = 2$

Answer: B

Solution:

$$a + \frac{b}{\log 2} = \int_2^e \left[\frac{1}{\log x} - \frac{1}{(\log x)^2}\right] dx$$

Put $\log x = t \Rightarrow \frac{1}{x} dx = dt \Rightarrow dx = e^t dt$

When $x = e, t = 1$ and when $x = 2, t = \log 2$

$$\therefore a + \frac{b}{\log 2} = \int_{\log 2}^{1} \left(\frac{1}{t} - \frac{1}{t^2}\right) e^t \cdot dt = \left[e^t \cdot \frac{1}{t}\right]_{\log 2}^{1} = e - \frac{e^{\log 2}}{\log 2} = e - \frac{2}{\log 2}$$

$$\therefore a = e, b = -2$$

Question113

$\int_0^{\pi/4} \log(1 + \tan x) dx =$ **MHT CET 2021 (20 Sep Shift 1)**

Options:

A. $\frac{\pi}{16} \log 2$

B. $\frac{\pi}{4} \log 2$

C. $\frac{\pi}{8} \log 2$

D. $\pi \log 2$

Answer: C

Solution:

$$\text{Let } I=\int_0^{\pi/4}\log(1+\tan x)dx$$
$$=\int_0^{\pi/4}\log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right]dx$$
$$=\int_0^{\pi/4}\log\left[1+\left(\frac{1-\tan x}{1+\tan x}\right)\right]dx=\int_0^{\pi/4}\log\left(\frac{2}{1+\tan x}\right)dx$$
$$=\int_0^{\pi/4}(\log 2)dx-\int_0^{\pi/4}\log(1+\tan x)dx=\int_0^{\pi/4}(\log 2)dx-I$$
$$\therefore 2I=(\log 2)[x]_0^{\pi/4}=(\log 2)\left(\frac{\pi}{4}\right)\Rightarrow I=\left(\frac{\pi}{8}\right)\log 2$$

Question114

$\int\tan^{-1}(\sec x+\tan x)dx=$ **MHT CET 2021 (20 Sep Shift 1)**

Options:

A. $\frac{\pi x}{4}+\frac{x^2}{4}+c$

B. $\sin x\cos x+c$

C. $\frac{\pi \mathrm{x}}{2}+\frac{\mathrm{x}^2}{2}+\mathrm{c}$

D. $\sin x+\cos x+c$

Answer: A

Solution:

$$\text{Let } I=\tan^{-1}(\sec x+\tan x)dx$$
$$=\int\tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)dx$$
$$=\int\tan^{-1}x\left[\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}\right]_{dx}$$
$$=\int\tan^{-1}\left(\frac{\cos\frac{x}{2}+\sin\frac{x}{2})}{\cos\frac{x}{2}-\sin\frac{x}{2}}\right)dx\quad=\int\tan^{-1}\left(\frac{1+\sin\frac{x}{2}}{1-\tan\frac{x}{2}}\right)dx$$
$$=\int\tan^{-1}\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right]dx=\int\left(\frac{\pi}{4}+\frac{x}{2}\right)dx$$
$$=\frac{\pi x}{4}+\frac{x^2}{4}+c$$

Question115

$\int_0^{\pi/2}\frac{dx}{5+4\sin x}=A\tan^{-1}B$, then $A+B=$ **MHT CET 2021 (20 Sep Shift 1)**

Options:

A. $\frac{2}{3}$

B. 1

C. 2

D. $\frac{1}{3}$

Answer: B

Solution:

Let $I=\int_0^{\pi/2} \frac{dx}{5+4\sin x}$ Put $\tan\frac{\mathrm{x}}{2}=\mathrm{t} \Rightarrow \sec^2\frac{\mathrm{x}}{2}\left(\frac{1}{2}\right)\mathrm{dx}=\mathrm{dt}$

$$\therefore \mathrm{dx}=\frac{2\mathrm{dt}}{1+\mathrm{t}^2} \text{ and } \sin\mathrm{x}=\frac{2\mathrm{t}}{1+\mathrm{t}^2}$$

When $\mathrm{x}=0, \mathrm{t}=0$ and when $\mathrm{x}=\frac{\pi}{2}, \mathrm{t}=1$

$$\therefore=\int_0^1 \frac{1}{5+4\left(\frac{2\mathrm{t}}{1+\mathrm{t}^2}\right)}\times\frac{2\mathrm{dt}}{1+\mathrm{t}^2}=2\int_0^1\frac{\mathrm{dt}}{5+5\mathrm{t}^2+8\mathrm{t}}=\frac{2}{5}\int_0^1\frac{\mathrm{dt}}{\mathrm{t}^2+\frac{8}{5}\mathrm{t}+1}$$

$$=\frac{2}{5}\int_0^1\frac{\mathrm{dt}}{\mathrm{t}^2+\frac{8}{5}\mathrm{t}+\frac{16}{25}+\frac{9}{25}}=\frac{2}{5}\int_0^1\frac{\mathrm{dt}}{\left(\mathrm{t}+\frac{4}{5}\right)^2+\left(\frac{3}{5}\right)^2}$$

$$\therefore I=\frac{2}{5}\times\frac{1}{\left(\frac{3}{5}\right)}\left[\tan^{-1}\left[\frac{\mathrm{t}+\frac{4}{5}}{\left(\frac{3}{5}\right)}\right]\right]_0^1=\frac{2}{3}\left[\tan^{-1}\left(\frac{5\mathrm{t}+4}{3}\right)\right]_0^1$$

$$=\frac{2}{3}\left[\tan^{-1}3-\tan^{-1}\frac{4}{3}\right]=\frac{2}{3}\tan^{-1}\left[\frac{3-\left(\frac{4}{3}\right)}{1+3\left(\frac{4}{3}\right)}\right]$$

$$=\frac{2}{3}\tan^{-1}\left(\frac{5}{3}\times\frac{1}{5}\right)=\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\right)$$

Comparing with given data, we get

$$\mathrm{A}=\frac{2}{3}, \mathrm{B}=\frac{1}{3}\Rightarrow\mathrm{A}+\mathrm{B}=1$$

Question116

$\int_0^4|x-2|dx=$ **MHT CET 2020 (20 Oct Shift 2)**

Options:

A. 0

B. 4

C. 8

D. 2

Answer: B

Solution:

$$\begin{aligned}\int_0^4|x-2|dx &= \int_0^2(2-x)dx+\int_2^4(x-2)dx\\ &= 2[x]_0^2-\frac{1}{2}\left[x^2\right]_0^2+\frac{1}{2}\left[x^2\right]_2^4-2[x]_2^4\\ &= 2(2)-\frac{1}{2}(4)+\frac{1}{2}(16-4)-2(4-2)\\ &= 4-2+6-4=4\end{aligned}$$

Question117

$\int_{-8}^{8}\frac{x^5+x^3}{4-x^2}dx=$ **MHT CET 2020 (20 Oct Shift 2)**

Options:

A. 16

B. 0

C. 8

D. -8

Answer: B

Solution:

Let $I=\int_{-8}^{8}\frac{x^5+x^3}{4-x^2}dx$

Let $f(x)=\frac{x^5+x^3}{4-x^2}\Rightarrow f(-x)=\frac{-(x^5+x^3)}{4-x^2}$

$\therefore\quad f(-x)=-f(x)\Rightarrow I=0$

Question118

If $\int_o^a\frac{dx}{1+4x^2}=\frac{\pi}{8}$**, then** $a=$ **MHT CET 2020 (20 Oct Shift 2)**

Options:

A. $\frac{1}{2}$

B. 2

C. $\frac{1}{4}$

D. 1

Answer: A

Solution:

We have $\int_0^a \frac{dx}{1+4x^2} = \frac{\pi}{8}$

$\therefore \frac{\pi}{8} = \frac{1}{4}\int_0^a \frac{dx^2+\left(\frac{1}{2}\right)^2}{2} = \frac{1}{4}\frac{1}{\left(\frac{1}{2}\right)}\left[\tan^{-1}\frac{x}{\left(\frac{1}{2}\right)}\right]_0^a$

$= \frac{1}{2}\left[\tan^{-1} 2a - \tan^{-1} 0\right]$

$\therefore \frac{\pi}{8} = \frac{1}{2}\tan^{-1} 2a \Rightarrow \frac{\pi}{4} = \tan^{-1} 2a \Rightarrow 2a = \tan\frac{\pi}{4} = 1 \Rightarrow a = \frac{1}{2}$

Question119

$\int_1^{28} \frac{dx}{x(1+\log x)^2}$ = **MHT CET 2020 (20 Oct Shift 1)**

Options:

A. $\cdot \log 2$

B. $1 + \log 2$

C. $\frac{\log 2}{(1+\log 2)}$

D. $\frac{1}{(1+\log 2)}$

Answer: C

Solution:

$I = \int_1^2 \frac{dx}{x(1+\log x)}$

Put $\quad 1 + \log x = t \Rightarrow \frac{1}{x}dx = dt$

When $x = 1, t = 1$ and when $x = 2, t = 1 + \log 2$

$$I = \int_1^{1+\log 2} \frac{dt}{t} = [\log t]_1^{1+\log 2} = \log(1+\log 2) - \log 1$$
$$= \log(1+\log 2)$$

Question120

$\int_0^{\pi} \frac{e^{\cos x}}{\left(e^{\cos x}+e^{-\cos x}\right)}dx$ = **MHT CET 2020 (20 Oct Shift 1)**

Options:

A. $\frac{-\pi}{2}$

B. $-\pi$

C. π

D. $\frac{\pi}{2}$

Answer: D

Solution:

$$\begin{aligned}
I &= \int_0^\pi \frac{e^{\cos x}}{e^{\cos x}+e^{-\cos x} dx} \ldots(1) \\
\text{Let} \quad &= \int_0^\pi \frac{e^{\cos(\pi-x)}}{e^{\cos(\pi-x)}+e^{-\cos(\pi-x)}} dx \\
&= \int_0^\pi \frac{e^{-\cos x}}{e^{-\cos x}+e^{\cos x} dx} \ldots(2)
\end{aligned}$$

Adding equation (1) \& (2), we get

$$2I = \int_0^\pi 1 dx = [x]_0^\pi \Rightarrow I = \frac{\pi}{2}$$

Question121

$\int_0^{\frac{\pi}{2}} \frac{\sqrt[7]{\sin x}}{\sqrt[7]{\sin x}+\sqrt[7]{\cos x}} dx =$ **MHT CET 2020 (19 Oct Shift 2)**

Options:

A. $\frac{\pi}{2}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{8}$

Answer: C

Solution:

$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt[7]{\sin x}}{\sqrt[7]{\sin x}+\sqrt[7]{\cos x}} \ldots(1)$

$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi-x}{2}\right)} dx}$

$I = \int_0^{\tau/2} \frac{\sqrt[7]{\cos x}}{\sqrt[7]{\cos x}+\sqrt[7]{\sin x}} dx \ldots(2)$

Equation $(1) - (2)$ gives

$$2I = \int_0^{\frac{\pi}{2}} 1 dx = [x]_0^{\frac{\pi}{2}} = \frac{\pi}{2}$$
$$\therefore I = \frac{\pi}{4}$$

Question122

$\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx =$ **MHT CET 2020 (19 Oct Shift 2)**

Options:

A. 1

B. 4

C. 2

D. 0

Answer: D

Solution:

$\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx$

$= \int_0^1 \tan^{-1}\left[\frac{x-(1-x)}{1+x(1-x)}\right] dx = \int_0^1 \left[\tan^{-1} x - \tan^{-1}(1-x)\right] dx$...(1)

$= \int_0^1 \left[\tan^{-1}(1-x) - \tan^{-1}(1-(1-x))\right] dx = \int_0^1 \left[\tan^{-1}(1-x) - \tan^{-1}(x)\right] dx$...(2)

Adding (1) \& (2), we get

$2I = 0 \Rightarrow I = 0$

Question123

$\int_0^1 \left(1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots \text{ upto } \infty\right) e^{2x} dx =$ **MHT CET 2020 (19 Oct Shift 2)**

Options:

A. e^2

B. $e - 1$

C. $e + 1$

D. e

Answer: B

Solution:

$\int_0^1 \left(1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots \infty\right) e^{2x} dx$

$= \int_0^1 e^{-x} e^{2x} dx = \int_0^1 e^x dx$

$= \left[e^x\right]_0^1 = e^1 - e^0 = e - 1$

Question124

$\int_0^1 x(1-x)^5 dx =$ **MHT CET 2020 (19 Oct Shift 1)**

Options:

A. $\frac{1}{7}$

B. $-\frac{1}{42}$

C. $\frac{1}{42}$

D. $\frac{1}{6}$

Answer: C

Solution:

$$\text{Let } I=\int_0^1 x(1-x)^5 dx$$
$$\therefore I=\int_0^1(1-x)(1-(1-x))^5 dx=\int_0^1(1-x)x^5 dx=\int\left(x^5-x^6\right) dx$$
$$=\left[\frac{x^6}{6}-\frac{x^7}{7}\right]_0^1=\frac{1}{6}-\frac{1}{7}=\frac{1}{42}$$

Question125

If $\int_1^k\left(3x^2+2x+1\right) dx=11$, then $\mathrm{k}=$ MHT CET 2020 (19 Oct Shift 1)

Options:

A. $\frac{1}{2}$

B. -2

C. $-\frac{1}{2}$

D. 2

Answer: D

Solution:

We have $\left[3\left(\frac{x^3}{3}\right)+2\left(\frac{x^2}{2}\right)+x\right]_1^k=11$

$\therefore\left[x^3+x^2+x\right]_1^k=11$

$\left(k^3+k^2+k\right)-(1+1+1)=11 \quad \Rightarrow k^3+k^2+k=14$

$\therefore \mathrm{k}\left(\mathrm{k}^2+\mathrm{k}+1\right)=2\times 7 \Rightarrow \mathrm{k}=2$

Question126

$\int_{-2}^{2.24}[x] dx=$ where $[x]$ is the greatest integer function MHT CET 2020 (19 Oct Shift 1)

Options:

A. 2

B. 4

C. -2

D. 0

Answer: C

Solution:

$$\int_{-2}^{2}[x]dx = \int_{-2}^{-1} -2dx + \int_{-1}^{0} -1dx + \int_{0}^{1} 0dx + \int_{1}^{2} 1dx$$
$$= -2[x]_{-2}^{-1} + (-1)[x]_{-1}^{0} + 1[x]_{1}^{2}$$
$$= -2[-1-(-2)] - 1[-(-1)] + 1[2-1]$$
$$= -2[1] - 1[1] + 1[1] = -2 - 1 + 1 = -2$$

Question127

$\int_{\frac{\pi}{5}}^{\frac{3\pi}{10}}\left[\frac{\tan x}{\tan x+\cot x}\right] dx =$ **MHT CET 2020 (16 Oct Shift 2)**

Options:

A. $\frac{\pi}{2}$

B. $\frac{3\pi}{10}$

C. $\frac{\pi}{5}$

D. $\frac{\pi}{20}$

Answer: D

Solution:

$$\int_{\frac{\pi}{5}}^{\frac{3\pi}{10}}\left[\frac{\tan x}{\tan x+\cot x}\right] dx \ldots(1)$$
$$= \int_{\frac{\pi}{5}}^{\frac{3\pi}{10}}\left[\frac{\tan\left(\frac{3\pi}{10}+\frac{\pi}{5}-x\right)+\cot\left(\frac{3\pi}{10}+\frac{\pi}{5}-x\right)}{\tan\left(\frac{3\pi}{10}+\frac{\pi}{5}-x\right)}\right] dx = \int_{\frac{\pi}{5}}^{10}\frac{3\pi}{\tan\left(\frac{\pi}{2}-x\right)+\cot\left(\frac{\pi}{2}-x\right)}\tan\left(\frac{\pi}{2}-x\right)$$
$$= \int_{\frac{\pi}{5}}^{\frac{3\pi}{10}}\frac{\tan x+\cot x}{d}x\ldots(2)$$

Equation (1) + (2) gives

$$2I = \int_{\frac{\pi}{5}}^{\frac{3\pi}{10}} dx = [x]_{\frac{\pi}{5}}^{\frac{3\pi}{10}} = \left(\frac{3\pi}{10}-\frac{\pi}{5}\right) = \frac{\pi}{10}$$
$$\therefore I = \frac{\pi}{20}$$

Question128

$\int_{-5}^{5}\log\left(\frac{7-x}{7+x}\right)dx =$ **MHT CET 2020 (16 Oct Shift 2)**

Options:

A. 5

B. 0

C. -5

D. 10

Answer: B

Solution:

Let $1 = \int_{-4}^{5} \log \frac{7-x}{7+x}$

Let $\quad f(x) = \log \frac{7-x}{7+x}$

$f(-x) = \log\left[\frac{7-(-x)}{7+(-x)}\right] = \log\left(\frac{7+x}{7-x}\right) = -\log\left(\frac{7-x}{7+x}\right) = -f(x)$

$\therefore f(x)$ is an odd function $\Rightarrow I = 0$

Question129

$\int_0^1 \left(\frac{x^2-2}{x^2+1}\right) dx =$ **MHT CET 2020 (16 Oct Shift 2)**

Options:

A. $1 + \frac{3\pi}{4}$

B. $1 - \frac{3\pi}{4}$

C. $1 - \frac{\pi}{4}$

D. $1 + \frac{\pi}{4}$

Answer: B

Solution:

$$\begin{aligned}\int_0^1 \frac{x^2+1-3}{x^2+1} dx &= \int_0^1 \frac{x^2+1}{x^2+1} - \frac{3}{x^2+1} dx \\ &= \int_0^1 1 - \frac{3}{x^2+1} dx = \left[x - 3\tan^{-1} x\right]_0^1 \\ &= \left(1 - 3\tan^{-1} 1\right) - \left(0 - 3\tan^{-1} 0\right) = 1 - 3\frac{\pi}{4}\end{aligned}$$

Question130

$\int_2^3 \frac{x}{x^2-1} dx =$ **MHT CET 2020 (16 Oct Shift 1)**

Options:

A. $\left(\frac{-1}{2}\right) \log\left(\frac{8}{3}\right)$

B. $\left(\frac{1}{2}\right) \log\left(\frac{8}{3}\right)$

C. $\left(\frac{-1}{3}\right) \log\left(\frac{8}{3}\right)$

D. $\left(\frac{1}{3}\right) \log\left(\frac{8}{3}\right)$

Answer: B

Solution:

$\int \frac{1}{x^2-1} dx$

$= \int_2^1 \frac{(x-1)+1}{(x-1)(x+1)} dx - \int_2^1 \frac{dx}{x+1} + \int_2^1 \left(\frac{1}{x-1} - \frac{1}{x+1}\right) \frac{1}{2} dx$

$= |\log|x+1||_2^1 + \frac{1}{2}\left[\log\left|\frac{x-1}{x+1}\right|\right|_2^1$

$= (\log 4 - \log 3) + \frac{1}{2}\left|\log\left(\frac{2}{4}\right) - \log\left(\frac{1}{3}\right)\right|$

$= \log\left(\frac{4}{3}\right) + \frac{1}{2} \mid \frac{\log\left(\frac{1}{2}\right)}{1}\Big) - \log\left(\frac{4}{3}\right) + \log\left|\frac{3}{2}\right|^2$

$= \log\left(\frac{4}{3} x \sqrt{\frac{3}{2}}\right)^1 - \log\left(\frac{8}{3}\right)^2 \mid -\frac{1}{2}\log\left(\frac{8}{3}\right)$

This problem con ulso be nolved an follows :

$\int_0^1 \frac{x}{x^2-1} dx$

$-\int_2^1 \frac{x}{(x-1)(x+1)} dx - \frac{1}{2}\int_0^1 \left|\frac{1}{x-1} + \frac{1}{x+1}\right| dx$

$= \frac{1}{2}\{[\log(x-1)]_2^3 + [\log(x+1)]_2^3\}$

$= \frac{1}{2}\{[\log[(x-1)(x+1)]]_2^3\} = \frac{1}{2}\left[\log(x^2-1)\right]_2^3$

$= \frac{1}{2}[\log(9-1) - \log(4-1)] = \frac{1}{2}\log\left(\frac{8}{3}\right)$

Question131

$\int_0^a \sqrt{\frac{x}{a-x}} dx =$ **MHT CET 2020 (16 Oct Shift 1)**

Options:

A. $\left(\frac{\pi}{4}\right) a$

B. $-\pi a$

C. $2\pi a$

D. $\left(\frac{\pi}{2}\right) a$

Answer: D

Solution:

$$I=\int_0^a \sqrt{\frac{x}{a-x}}\,dx$$

Let $x=a\sin^2\theta \Rightarrow dx=2a\sin\theta\cos\theta\,d\theta$, and when $x:0\to a,\ \theta:0\to\frac{\pi}{2}$.

Then

$$\sqrt{\frac{x}{a-x}}=\sqrt{\frac{a\sin^2\theta}{a\cos^2\theta}}=\tan\theta.$$

So

$$I=\int_0^{\pi/2}\tan\theta\,(2a\sin\theta\cos\theta)\,d\theta=2a\int_0^{\pi/2}\sin^2\theta\,d\theta=2a\cdot\frac{\pi}{4}=\frac{\pi a}{2}.$$

Answer: $\boxed{\frac{\pi}{2}a}$.

Question132

$\int_0^{\frac{\pi}{2}}\log\left[\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}\right]dx=$ **MHT CET 2020 (16 Oct Shift 1)**

Options:

A. 1

B. $\frac{\pi}{4}$

C. 0

D. $\frac{\pi}{8}$

Answer: C

Solution:

$\int_0^{\frac{\pi}{2}}\log\sqrt{\frac{2\sin^2 x}{2\cos^2 x}}dx$

$=\int_0^{\pi/2}\log\left(\frac{\sin x}{\cos x}\right)dx\Rightarrow\int_0^{\pi/2}\log(\tan x)dx$...(1)

$=\int_0^{\pi/2}\left[\tan\left(\frac{\pi}{2}-x\right)\right]dx=\int_0^{\pi/2}\log\cot x dx$...(2)

Eq. (1) $+(2)$ gives

$$2\mathrm{I}=\int_0^{\pi/2}[\log(\tan\mathrm{x})+\log(\cot\mathrm{x})]\mathrm{dx}=\int_0^{\pi/2}\log[\tan\mathrm{x}\cot\mathrm{x}]\mathrm{dx}$$
$$=\int_0^{\pi/2}\log 1\mathrm{dx}=0\Rightarrow\mathrm{I}=0$$

Question133

$\int_0^{\frac{\pi}{2}}\sin^2 x\ \mathrm{d}x=$ **MHT CET 2020 (15 Oct Shift 2)**

Options:

A. $\frac{\pi}{2}$

B. $\frac{3\pi}{2}$

C. $\frac{3\pi}{4}$

D. $\frac{\pi}{4}$

Answer: D

Solution:

$$\int_0^{\frac{\pi}{2}} \sin^2 x dx = \frac{1}{2} \int_0^{\frac{\pi}{2}} (1 - \cos 2x) dx = \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}}$$
$$= \frac{1}{2}\left[\frac{\pi}{2} + 0\right] = \frac{\pi}{4}$$

Question134

$\int_{-4}^{4} \log\left(\frac{8-x}{8+x}\right) dx =$ **MHT CET 2020 (15 Oct Shift 2)**

Options:

A. -4

B. 8

C. 4

D. 0

Answer: D

Solution:

$$\begin{aligned} \text{Let } f(x) &= \log(8-x) - \log(8+x) \\ f(-x) &= \log(8+x) - \log(8-x) \\ &= -[\log(8-x) - \log(8+x)] \\ \text{Thus } f(-x) &= -f(x) \Rightarrow I = 0 \end{aligned}$$

Question135

$\int_0^5 \frac{dx}{x^2+2x+10}$ = **MHT CET 2020 (15 Oct Shift 2)**

Options:

A. $\frac{\pi}{6}$

B. $\frac{\pi}{12}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{4}$

Answer: B

Solution:

$$I=\int_0^5 \frac{dx}{x^2+2x+10}=\int_0^5 \frac{dx}{(x+1)^2+(3)^2}$$
$$=\frac{1}{3}\left[\tan^{-1}\frac{x+1}{3}\right]_0^5=\frac{1}{3}\left[\tan^{-1}2-\tan^{-1}\frac{1}{3}\right]$$
$$=\frac{1}{3}\left[\tan^{-1}\left[\frac{2-\frac{1}{3}}{1+\frac{2}{3}}\right]\right]=\frac{1}{3}\tan^{-1}(1)$$
$$=\frac{1}{3}\times\frac{\pi}{4}=\frac{\pi}{12}$$

Question136

$\int_{-2}^{1}[x+1]dx=$**(Where $[x]$ is greatest integer function not greater than x) MHT CET 2020 (15 Oct Shift 1)**

Options:

A. 1

B. 0

C. -1

D. 2

Answer: B

Solution:

$$\int_{-2}^{1}[x+1]dx=\int_{-2}^{-1}([x]+1)dx+\int_{-1}^{0}([x]+1)dy+\int_{0}^{1}([x]+1)dx$$
$$=\int_{-2}^{-1}(-2+1)dx+\int_{-1}^{0}(-1+1)dx+\int_{0}^{1}(0+1)dx$$
$$=-[x]_{-2}^{-1}+0+[x]_0^1=-(-1+2)+0+(1-0)$$
$$=0$$

Question137

$\int_0^{\frac{\pi}{2}}\left(e^{\sin x}-e^{\cos x}\right)dx=$ **MHT CET 2020 (15 Oct Shift 1)**

Options:

A. $\frac{1}{2}$

B. 0

C. 1

D. $\frac{\pi}{4}$

Answer: B

Solution:

$$I=\int_0^{\frac{\pi}{2}}\left(e^{\sin x}-e^{\cos x}\right) dx \ldots .(1)$$

$$\text{Let } =\int_0^{\pi / 2} e^{\sin \left(\frac{\pi}{2}-x\right)}-e^{\cos \left(\frac{\pi}{2}-x\right)} \mathrm{dx}$$

$$=\int_0^{\pi / 2} e^{\cos x}-e^{\sin x} d x \ldots .(2)$$

Adding equation (1) \& (2) we get

$$2 I=\int_0^{\pi / 2}\left(e^{\sin x}-e^{\cos x}-e^{\cos x}-e^{\sin x}\right) d x$$
$$2 I=0 \Rightarrow I=0$$

Question138

$\int_0^1 \frac{x^2}{1+x^2} d x=$ **MHT CET 2020 (15 Oct Shift 1)**

Options:

A. $1+\frac{\pi}{4}$

B. $1-\frac{\pi}{4}$

C. $1-\frac{\pi}{2}$

D. $1+\frac{\pi}{2}$

Answer: B

Solution:

Let

$$I=\int_0^1 \frac{x^2}{1+x^2} d x=\int_0^1 \frac{\left(1+x^2\right)-1}{1+x^2} d x=\int_0^1\left(\frac{1+x^2}{1+x^2}-\frac{1}{1+x^2}\right) d x=\int_0^1\left(1-\frac{}{1+}\right.$$
$$=\left[x-\tan ^{-1} x\right]_0^1=1-\frac{\pi}{4}$$

Question139

$$\int_0^\pi \frac{x \cos x \cdot \sin x}{\cos ^3 x+\cos x} d x=$$

MHT CET 2020 (14 Oct Shift 2)

Options:

A. $\frac{\pi}{4}$

B. $\frac{\pi^2}{4}$

C. $\frac{\pi}{8}$

D. $\frac{\pi^2}{8}$

Answer: B

Solution:

Let I $= \int_0^\pi \frac{x \cos x \sin x}{\cos^3 x + \cos x} dx$

$= \int_0^\pi \frac{x \sin x}{\cos^2 x + 1} dx$

$= \int_0^\pi \frac{(\pi - x) \sin x}{\cos^2 x + 1} dx = \int_0^\pi \frac{\pi \sin x}{\cos^2 x + 1} dx - \int_0^\pi \frac{x \sin x}{\cos^2 x + 1} dx$

$= \int_0^\pi \frac{\pi \sin x}{\cos^2 x + 1} dx - I$

$2I = \int_0^\pi \frac{\pi \sin x}{\cos^2 x + 1} dx$

Put $\cos x = t = \sin dx = -dt$

When $x = 0, t = 1$ and when $x = \pi, t = -1$

$2I = -\int_1^{-1} \frac{\pi dt}{1+t^2} = \pi \int_{-1}^1 \frac{dt}{1+t^2} = 2\pi \int_0^1 \frac{dt}{1+t^2}$

$2I = 2\pi \left[\tan^{-1} t\right]_0^1 = 2\pi \left(\frac{\pi}{4}\right) = \left(\frac{\pi^2}{2}\right)$

$\therefore \frac{\pi^2}{4}$

Question140

$$\int_0^{\frac{\pi}{2}} \frac{dx}{1 + \cos x} =$$

MHT CET 2020 (14 Oct Shift 2)

Options:

A. -2

B. 2

C. 1

D. -1

Answer: C

Solution:

$$\text{Let1} = \int_0^{\frac{\pi}{2}} \frac{\text{dx}}{1 + \cos \text{x}} = \int_0^{\frac{\pi}{2}} \frac{\text{dx}}{2 \cos^2 \frac{\text{x}}{2}} = \frac{1}{2} \int_0^{\frac{\pi}{2}} \sec^2 \frac{\text{x}}{2} \text{dx}$$

$$= \frac{1}{2} \left[\frac{\tan \frac{\text{x}}{2}}{\left(\frac{1}{2}\right)} \right]_0^{\frac{\pi}{2}} = \tan \frac{\pi}{4} - \tan 0 = 1 - 0 = 1$$

Question141

$$\int_{-1}^{1}\left[\sqrt{1+x+x^2}-\sqrt{1-x+x^2}\right]dx =$$

MHT CET 2020 (14 Oct Shift 2)

Options:

A. 2

B. 5

C. 1

D. 0

Answer: D

Solution:

Given I $= \int_{-1}^{1}\left(\sqrt{1+x+x^2}-\sqrt{1-x+x^2}\right)dx$

Let $f(x)=\sqrt{1+x+x^2}-\sqrt{1-x+x^2}$

$\therefore f(-x)=\sqrt{1-x+x^2}-\sqrt{1+x+x^2}=-\left(\sqrt{1+x+x^2}-\sqrt{1-x-x^2}\right)=-f(x)$

$\therefore I=0$

Question142

If $\int_0^1\left(5x^2-3x+k\right)dx=0$, then $k=$ MHT CET 2020 (14 Oct Shift 1)

Options:

A. $\frac{1}{3}$

B. $\frac{1}{6}$

C. $\frac{-1}{3}$

D. $\frac{-1}{6}$

Answer: D

Solution:

Given $\int_0^1\left(5x^2-3x+k\right)dx=0$

$\therefore\left[5\frac{x^3}{3}-\frac{3x^2}{2}+kx\right]_0^1=0\Rightarrow\frac{5}{3}-\frac{3}{2}+k=0\Rightarrow k=-\frac{1}{6}$

Question143

$\int_{-\pi}^{\pi}\frac{2x}{1+\cos^2 x}dx=$ **MHT CET 2020 (14 Oct Shift 1)**

Options:

A. π

B. 0

C. 1

D. $-\pi$

Answer: B

Solution:

Let $f(x)=\frac{2x}{1+\cos^2 x} \Rightarrow f(-x)=\frac{-2x}{1+\cos^2 x}$

Thus $\mathrm{f}(-\mathrm{x})=-\mathrm{f}(\mathrm{x}) \Rightarrow \mathrm{I}=0$

Question144

$\frac{d^2y}{dx^2}=\sin x+e^x; y(0)=3$ and $\frac{dy}{dx}$ at $x=0$ is 4 , then the equation of the MHT CET 2020 (14 Oct Shift 1)

Options:

A. $y=4+2x+e^x-\sin x$

B. $y=2+3x+e^x-\sin x$

C. $y=2+4x+e^x-\sin x$

D. $y=4+2x+e^x+\sin x$

Answer: C

Solution:

$\frac{d}{dx}\left(\frac{dy}{dx}\right)=\sin x+e^x$

Integrating both sides.;

$y \rightarrow \frac{dy}{dx}=e^x-\cos x+c$

at $x=0; \frac{dy}{dx}=4; \quad c=4$

$\frac{dy}{dx}=e^x-\cos x+4$

Integrating both sides;

$y=e^x-\sin x+4x+c$

put $x=0; y(0)=3$;

$c=2$

Equation of curve $y=e^x-\sin x+4x+2$

Question145

$\int_0^1 \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx=$ MHT CET 2020 (14 Oct Shift 1)

Options:

A. $\pi-\log 2$

B. $\frac{\pi}{2} - \log 2$

C. $\pi + \log 2$

D. $\frac{\pi}{2} + \log 2$

Answer: B

Solution:

Let $I = \int_0^1 \tan^{-1}\left(\frac{2x}{1-x^2}\right) dx$

Put $x = \tan\theta \Rightarrow dx = \sec^2\theta d\theta$

When $x = 0, \theta = 0$ and when $x = 1, \theta = \frac{\pi}{4}$

$\therefore \mathrm{I} = \int_0^{\frac{\pi}{4}} \left[\tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)\right] (1 + \tan^2\theta)\, \mathrm{d}\theta = \int_0^{\frac{\pi}{4}} \tan^{-1}(\tan 2\theta)\,(1 + \tan^2\theta)\, \mathrm{d}\theta$

$= \int_0^{\frac{\pi}{4}} 2\theta \sec^2\theta d\theta = 2\int_0^{\pi/4} \theta \sec^2\theta d\theta = 2[\theta\tan\theta]_0^{\pi/4} - 2\int_0^{\pi/4} \tan\theta d\theta$

$= 2\left[\frac{\pi}{4}\right] + 2[\log|\cos\theta|]_0^{\pi/4} = \frac{\pi}{2} + 2\left[\log\left|\frac{1}{\sqrt{2}}\right|\right]$

$= \frac{\pi}{2} + 2\log(2)^{\frac{1}{2}}$

$= \frac{\pi}{2} - \log 2$

Question146

$\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16\sin 2x} dx = k\log 3$, **then** $k =$ **MHT CET 2020 (13 Oct Shift 2)**

Options:

A. $\frac{1}{30}$

B. $\frac{1}{20}$

C. $\frac{1}{10}$

D. $\frac{1}{40}$

Answer: B

Solution:

Let $I=\int_0^{\frac{\pi}{4}} \frac{\sin x+\cos x}{9+16 \sin 2x} dx=K \log 3$

Put $\sin x-\cos x=t \Rightarrow(\cos x+\sin x) dx=dt$ ...(1)

Now squaring equation (1), we get

$$\begin{aligned} \therefore I & =\int_{-1}^0 \frac{dt}{9+16\left(1-t^2\right)} \\ & =\int_{-1}^0 \frac{dt}{25-16 t^2}=\frac{1}{16} \int_{-1}^0 \frac{dt}{\left(\frac{5}{4}\right)^2-t^2} \end{aligned}$$

$$=\frac{1}{16} \times \frac{1}{2\left(\frac{5}{4}\right)}\left[\log \mid \frac{\frac{5}{4}+t}{\frac{5}{4}-t}\right]_{-1}^0=\frac{1}{40}\left[\log \left(\frac{5+4t}{5-4t}\right)\right]_{-1}^0=\frac{1}{40}\left[\log (1)-\log \left(\frac{1}{9}\right)\right]$$
$$=\frac{1}{40}(\log 9)=\frac{1}{40} \log 3^2=\frac{2}{40} \log 3=\frac{1}{20} \log 3$$

As per given data, $K=\frac{1}{20}$

Question147

$\int_0^{\frac{\pi}{2}} \frac{1-\cot x}{\operatorname{cosec} x+\cos x} dx=$ **MHT CET 2020 (13 Oct Shift 2)**

Options:

A. 0

B. $\frac{\pi}{2}$

C. 1

D. $\frac{\pi}{4}$

Answer: A

Solution:

Let
$$\begin{aligned} & I=\int_0^{\frac{\pi}{2}} \frac{1-\cot x}{\operatorname{cosec} x+\cos x} dx \\ & I=\int_0^{\frac{\pi}{2}} \frac{1-\frac{\cos x}{\sin x}}{\frac{1}{\sin x}+\cos x} dx=\int_0^{\frac{\pi}{2}} \frac{\sin x-\cos x}{1+\sin x \cos x} dx \ldots(1) \\ & I=\int_0^{\frac{\pi}{2}} \frac{\sin \left(\frac{\pi}{2}-x\right)-\cos \left(\frac{\pi}{2}-x\right)}{1+\sin \left(\frac{\pi}{2}-x\right) \cos \left(\frac{\pi}{2}-x\right)} dx \\ & I=\int_0^{\frac{\pi}{2}} \frac{\cos x-\sin x}{1+\cos x \sin x} dx \ldots(2) \end{aligned}$$

Adding equations (1) \& (2), we get

$2I=\int_0^{\frac{\pi}{2}} \frac{\sin x-\cos x+\cos x-\sin x}{1+\sin x \cos x} dx$
$2I=0 \Rightarrow I=0$

Question148

$\int_0^{\frac{\pi}{2}} \frac{\sqrt[3]{\sec x}}{\sqrt[3]{\sec x}+\sqrt[3]{\operatorname{cosec} x}} dx=$ **MHT CET 2020 (13 Oct Shift 2)**

Options:

A. 0

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. $\frac{-\pi}{4}$

Answer: B

Solution:

Let

$$I=\int_0^{\frac{\pi}{2}} \frac{\sqrt[3]{\sec x}}{\sqrt[3]{\sec x}+\sqrt[3]{\operatorname{cosec} x}} \mathrm{dx} \ldots(1)$$

$$=\int_0^{\frac{\pi}{2}} \frac{\sqrt[3]{\sec \left(\frac{\pi}{2}-x\right)} \sqrt{\sec \left(\frac{\pi}{2}-x\right)+\sqrt[3]{\operatorname{cosec}\left(\frac{\pi}{2}-x\right)}}}{\sqrt[3]{\frac{\pi}{2}}} d x$$

$$\therefore \mathrm{I}=\int_0^{\frac{\pi}{2}} \frac{\sqrt[3]{\sec x}+\sqrt[3]{\operatorname{cosec} x}}{d} x \ldots(2)$$

Adding equation (1) \& (2), we get

$$2 I=\int_0^{\frac{\pi}{2}} 1 d x=[x]_0^{\frac{\pi}{2}} \Rightarrow 2 I=\frac{\pi}{2}-0 \Rightarrow I=\frac{\pi}{4}$$

Question149

$$\int_{-a}^a x^2\left(\frac{e^{x^3}-e^{-x^3}}{e^{x^3}+e^{-x^3}}\right) d x=$$

MHT CET 2020 (13 Oct Shift 1)

Options:

A. a^2

B. 0

C. a

D. $2 \int_0^a x^2\left(\frac{e^{x^3}-e^{-x^3}}{e^{x^3}+e^{-x^3}}\right) d x$

Answer: B

Solution:

Let

$$f(x)=x^2\left[\frac{e^{x^3}-e^{-x^3}}{e^{x^3}+e^{-x^3}}\right]=x^2\left[\frac{e^{x^3}-\frac{1}{e^{x^3}}}{e^{x^3}+\frac{1}{e^{x^3}}}\right]=x^2\left[\frac{\left(e^{x^3}\right)^2-1}{\left(e^{x^3}\right)^2+1}\right]$$

$$f(-x)=(-x)^2\left[\frac{e^{-x^3}-e^{x^3}}{e^{-x^3}+e^{x^3}}\right]$$

$$=x^2\left[\frac{\frac{1}{e^{x^3}}-e^{x^3}}{\frac{1}{e^{x^3}}+e^{x^3}}\right]=x^2\left[\frac{1-\left(e^{x^3}\right)^2}{1+\left(e^{x^3}\right)^2}\right]=-x^2\left[\frac{\left(e^{x^3}\right)^2-1}{1+\left(e^{x^3}\right)2}\right]=-f(x)$$

Thus $f(-x)=-f(x)\Rightarrow$ Given function is an odd function.

$\therefore \quad I=0$

Question150

$\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{2}{3}} x}{\sin^{\frac{2}{3}} x+\cos^{\frac{2}{3}} x} dx=$ **MHT CET 2020 (13 Oct Shift 1)**

Options:

A. $\frac{\pi}{4}$

B. $\frac{\pi}{8}$

C. $\frac{\pi}{2}$

D. π

Answer: A

Solution:

$$\begin{aligned} I&=\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{2}{3}} x}{\sin^{\frac{2}{3}} x+\cos^{\frac{2}{3}} x} dx \ldots(1) \\ &=\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{2}{3}}\left(\frac{\pi}{2}-x\right)}{\sin^{\frac{2}{3}}\left(\frac{\pi}{2}-x\right)+\cos^{\frac{2}{3}}\left(\frac{\pi}{2}-x\right)} dx \\ \therefore I&=\int_0^2 \frac{\cos^{\frac{2}{3}} x}{\sin^{\frac{2}{3}} x+\cos^{\frac{2}{3}} x} dx \ldots(2) \end{aligned}$$

Adding equation (1) \& (2) we get

$$\begin{aligned} 2I&=\int_0^2 1 dx \Rightarrow 2I=[x]_0^{\frac{\pi}{2}} \\ I&=\frac{1}{2}\left(\frac{\pi}{2}-0\right)=\frac{\pi}{4} \end{aligned}$$

Question151

$\int_0^1 \tan^{-1}\left[\frac{2x-1}{1+x-x^2}\right] dx=$ **MHT CET 2020 (13 Oct Shift 1)**

Options:

A. 0

B. $\frac{\pi}{6}$

C. 1

D. $\frac{\pi}{4}$

Answer: A

Solution:

Let

$$I=\int_0^1 \tan^{-1}\left[\frac{2x-1}{1+x-x^2}\right]dx$$
$$=\int_0^1 \tan^{-1}\left[\frac{2x-1}{1+x(1-x)}\right]dx=\int_0^1 \tan^{-1}\left[\frac{x-(1-x)}{1+x(1-x)}\right]dx$$
$$=\int_0^1 \left[\tan^{-1}x-\tan^{-1}(1-x)\right]dx\ldots(1)$$
$$=\int_0^1 \left\{\tan^{-1}(1-x)-\tan^{-1}[1-(1-x)]\right\}dx$$
$$=\int_0^1 \left[\tan^{-1}(1-x)-\tan^{-1}x\right]dx\ldots(2)$$

Equation (1) + (2) gives

$$2I=0\Rightarrow I=0$$

Question152

$\int_0^a (a-x)^{\frac{3}{2}}\cdot x^2dx=$ **MHT CET 2020 (12 Oct Shift 2)**

Options:

A. $\frac{-16a^{\frac{9}{2}}}{315}$

B. $\frac{16a^{\frac{9}{2}}}{315}$

C. $\frac{16a^{\frac{7}{2}}}{315}$

D. $\frac{-16a^{\frac{7}{2}}}{315}$

Answer: B

Solution:

$$I=\int_0^a (a-x)^{\frac{3}{2}} x^2 dx$$

$$=\int_0^a [a-(a-x)]^{\frac{3}{2}}(a-x)^2 dx$$

$$=\int_0^a x^{\frac{3}{2}}\left(a^2-2ax+x^2\right) dx=\int_0^a \left(a^2x^{\frac{3}{2}}-2ax^{\frac{5}{2}}+x^{\frac{7}{2}}\right)_{dx}$$

Let

$$=a^2\left[\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^a-2a\left[\frac{x^{\frac{7}{2}}}{\frac{7}{2}}\right]_0^a+\left[\frac{x^{\frac{9}{2}}}{\frac{9}{2}}\right]_0^a$$

$$=\frac{2a^2}{5}\left[a^{\frac{5}{2}}-0\right]-2a\times\frac{2}{7}\left[a^{\frac{7}{2}}-0\right]+\frac{2}{9}\left[a^{\frac{9}{2}}-0\right]=\frac{2}{5}a^{\frac{9}{2}}-\frac{4}{7}a^{\frac{9}{2}}+\frac{2}{9}a^{\frac{9}{2}}$$

$$=a^{\frac{9}{2}}\left[\frac{2}{5}-\frac{4}{7}+\frac{2}{9}\right]=\frac{16}{315}a^{\frac{9}{2}}$$

Question153

$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^2 x dx=$ **MHT CET 2020 (12 Oct Shift 2)**

Options:

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. $\frac{3\pi}{4}$

Answer: C

Solution:

Let $f(x)=\sin^2 x$

$\therefore \mathrm{f}(-\mathrm{x})=[\sin(-\mathrm{x})]^2=\sin^2 \mathrm{x}$

Thus $\sin^2 x$ is an even function.

$\therefore \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^2 x dx=2\int_0^{\frac{\pi}{2}} \sin^2 x dx$

$=2\int_0^{\frac{\pi}{2}} \frac{1}{2}(1-\cos 2x) dx=\left[x-\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}}$

$=\left(\frac{\pi}{2}-0\right)-\left(\frac{\sin\pi}{2}-\frac{\sin 0}{2}\right)=\frac{\pi}{2}$

Question154

If the body cools from $135^\circ\mathrm{C}$ to $80^\circ\mathrm{C}$ at room temperature of $25^\circ\mathrm{C}$ in 60 minutes,then the temperature of body after 2 hours is MHT CET 2020 (12 Oct Shift 1)

Options:

A. $(52.5)^{\circ}\mathrm{C}$

B. $(10.5)^{\circ}\mathrm{C}$

C. $(52.75)^{\circ}\mathrm{C}$

D. $(10.75)^{\circ}\mathrm{C}$

Answer: A

Solution:

Let $\theta^{\circ}\mathrm{C}$ be the temperature of the body at time t min. Room temp is given $25^{\circ}\mathrm{C}$ Then by Newton's law of cooling, we write

$$\frac{d\theta}{dt} \propto (\theta - 25) \Rightarrow \frac{d\theta}{dt} = -k(\theta - 25)$$
$$\therefore \int \frac{d\theta}{\theta - 25} = \int -k dt$$
$$\log(\theta - 25) = -kt + c \ldots (1)$$

Initially when $t = 0, \theta = 135$

$$\therefore \log(135 - 25) = 0 + c \Rightarrow c = \log 110$$
$$\log(\theta - 25) = -kt + \log 110$$
$$\therefore \log\left(\frac{\theta - 25}{110}\right) = -kt \ldots (2)$$

Now when $t = 60, \theta = 80$

w when $t = 60, \theta = 80$
$\log\left(\frac{55}{110}\right) = -60k \Rightarrow k = -\frac{1}{60}\log\left(\frac{1}{2}\right)$

From (2) $\log\left(\frac{\theta-25}{110}\right) = \frac{t}{60}\log\left(\frac{1}{2}\right)$

At $t = 120$, we get

$$\log\left(\frac{\theta - 25}{110}\right) = 2\log\frac{1}{2} \Rightarrow \log\left(\frac{\theta - 25}{110}\right) = \log\left(\frac{1}{4}\right)$$

$\therefore \frac{\theta-25}{110} = \frac{1}{4} \Rightarrow 4\theta - 100 = 110 \Rightarrow 4\theta = 210 \Rightarrow \theta = 52.5^{\circ}\mathrm{C}$

Question155

$\int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{1+\sin^4 x} dx =$ **MHT CET 2020 (12 Oct Shift 1)**

Options:

A. $\frac{\pi}{6}$

B. $\frac{\pi}{8}$

C. $\frac{\pi}{2}$

D. $\frac{\pi}{4}$

Answer: B

Solution:

$I=\int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{1+\sin ^4 x} d x \quad=\frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{2 \sin x \cos x}{1+\left(\sin ^2 x\right)^2} d x$

Put $\sin ^2 x=t \Rightarrow 2 \sin x \cos x d x=d t$

When $x=0, t=0$ and when $x=\frac{\pi}{2}, t=1$

$\therefore I=\frac{1}{2} \int_0^1 \frac{\mathrm{dt}}{1+\mathrm{t}^2}=\frac{1}{2}\left[\tan ^{-1} \mathrm{t}\right]_0^1=\frac{1}{2}\left[\frac{\pi}{4}-0\right]=\frac{\pi}{8}$

Question156

$\int_{-5}^5\left[\frac{e^x+e^{-x}}{e^x-e^{-x}}\right] d x=$ **MHT CET 2020 (12 Oct Shift 1)**

Options:

A. 0

B. 1

C. $3 e^5$

D. $2 e^5$

Answer: A

Solution:

$$\begin{aligned} & \text{Let } f(x)=\frac{e^x+e^{-x}}{e^x-e^{-x}} \\ & =\frac{e^x+\frac{1}{e^x}}{e^x-\frac{1}{e^x}}=\frac{e^{2 x}+1}{e^{2 x}-1} \text{ and } f(-x)=\frac{e^{-x}+e^x}{e^{-x}-e^x}=\frac{\frac{1}{e^x}+e^x}{\frac{1}{e^x}-e^x}=\frac{1+e^{2 x}}{1-e^{2 x}} \\ & \therefore f(x)=-f(x)\end{aligned}$$

Thus $f(x)$ is an odd function.

$\therefore \int_{-5}^5 \frac{e^x+e^{-x}}{e^x-e^{-x}} d x=0$

Question157

The value of $\int_{-3}^{3}\left(a x^5+b x^3+c x+k\right) d x$**, where** a, b, c, k **are constants, depends only on ________ MHT CET 2019 (02 May Shift 1)**

Options:

A. a, b and c

B. k

C. a and b

D. a and k

Answer: B

Solution:

Let $I=\int_{-3}^{3}\left(ax^5+bx^3+cx+k\right)dx$

Where, a, b, c and k are constants.

$=\int_{-3}^{3}\left(ax^5+bx^3+cx\right)dx+\int_{-3}^{3}kdx$

$(\because ax^5+bx^3+cx$ is an odd function)

$\Rightarrow I=0+2\int_0^3 kdx$

Question158

$\int_0^4 \frac{1}{1+\sqrt{x}}dx=$ ________ **MHT CET 2019 (02 May Shift 1)**

Options:

A. $\log\left(\frac{e^4}{6}\right)$

B. $\log\left(\frac{e^4}{3}\right)$

C. $\log\left(\frac{e^4}{9}\right)$

D. $\log\left(\frac{e^3}{4}\right)$

Answer: C

Solution:

Let $I=\int_0^4 \frac{dx}{1+\sqrt{x}}$ put $x=t^2$

$dx=2tdt$

Then, $I=\int_0^2 \frac{2tdt}{1+t}=2\int_0^2 \frac{(1+t)-1}{1+t}dt$

$I=2\int_0^2\left(1-\frac{1}{1+t}\right)dt$

$I=2(2-\ln 3)=\log\left(\frac{e^4}{9}\right)$

Question159

$\int_a^b \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a+b-x}}dx=$......... **MHT CET 2019 (Shift 2)**

Options:

A. $a+b$

B. $\frac{b-a}{2}$

C. $a-b$

D. $\frac{a-b}{2}$

Answer: B

Solution:

Let $l=\int_a^b \frac{\sqrt{x}}{a\sqrt{x}+\sqrt{a+b-x}}dx$ ....(i)

$l=\int_a^b \frac{\sqrt{a+b-x}}{\sqrt{a+b-x}+\sqrt{a+b-(a+b-x)}}$

$l=\int_a^b \frac{\sqrt{a+b-x}}{\sqrt{a+b-x}+\sqrt{x}}$ ....(ii)

On adding Eqs. (i) and (ii), we get

$2l=\int_a^b \frac{\sqrt{x}+\sqrt{a+b-x}}{\sqrt{x}+\sqrt{a+b-x}}dx$

$\Rightarrow 2l=\int_a^b dx=(x)_a^b=b-a$

$\Rightarrow l=\frac{b-a}{2}$

Question160

$\int_0^1 x(1-x)^5 dx=\ldots\ldots.$ **MHT CET 2019 (Shift 2)**

Options:

A. $\frac{1}{5}$

B. $\frac{1}{42}$

C. $\frac{1}{13}$

D. $\frac{13}{42}$

Answer: B

Solution:

Let $= l=\int_0^1 x(1-x)^5 dx$

Put $1-x=t \Rightarrow x=1-t$

$\Rightarrow -dx=dt$

$\Rightarrow dx=-dt$

When $x=0'$, then $t=1$ and

When $x=1, thent=0$

$\therefore l\int_1^0(1-t)(t)^5(-dt)=\int_0^1\left(t^5-t^6\right)dt$

$=\left(\frac{t^6}{6}-\frac{t^7}{7}\right)=\frac{1}{6}(1)^6-\frac{1}{7}(1)^7$

$=\frac{7-6}{42}=\frac{1}{42}$

Question161

If $\int_0^a \sqrt{\frac{a-x}{x}}dx=\frac{K}{2}$ **, then K = …. MHT CET 2019 (Shift 2)**

Options:

A. $\frac{\pi a}{2}$

B. $\frac{5\pi a}{2}$

C. $\frac{3\pi a}{2}$

D. πa

Answer: D

Solution:

We have, $\int_0^a \sqrt{\frac{a-x}{x}} dx = \frac{k}{2}$
Let $l = \int_0^a \sqrt{\frac{a-x}{x}} dx$
Put $x = a\sin^2\theta$
$\Rightarrow dx = a(2\sin\theta\cos\theta)d\theta$
When, $x = 0, \theta = 0 and x = a, \theta = \frac{\pi}{2}$
$\therefore \int\limits_0^{\pi/2} \sqrt{\frac{a-a\sin^2\theta}{a\sin^2\theta}}(2a\sin\theta\cos\theta)d\theta = 2a\int\limits_0^{\pi/2}(cot\theta)(\sin\theta\cos\theta)d\theta$
$= 2a\int\limits_0^{\pi/2}\cos^2\theta d\theta = 2a\int_0^{\pi/2}\frac{1+\cos2\theta}{2}d\theta$
$= a\int_0^{\pi/2}(1+\cos2\theta)d\theta$
$= a\left[\left(\theta + \frac{\sin2\theta}{2}\right)\right]_0^{\pi/2} = a\left[\frac{\pi}{2} + \frac{\sin\pi}{2}\right] = \frac{\pi a}{2}$
$\therefore k = \pi a$

Question162

$\int_0^{\frac{\pi}{2}} \sqrt{\cos\theta}.\sin^3\theta d\theta = \ldots$ **MHT CET 2019 (Shift 1)**

Options:

A. $\frac{-20}{21}$

B. $\frac{-8}{21}$

C. $\frac{20}{21}$

D. $\frac{8}{21}$

Answer: D

Solution:

Let $l = \int_0^{\frac{\pi}{2}} \sqrt{\cos\theta}.\sin^3\theta d\theta$
$= \int_0^{\frac{\pi}{2}} \sqrt{\cos\theta}.\sin\theta\left(1-\cos^2\theta\right)d\theta$
Put $\cos\theta = t$
$\Rightarrow -\sin\theta d\theta = dt$
$\Rightarrow \sin\theta d\theta = -dt$

If $\theta=0, t=1\, and\, \theta=\frac{\pi}{2}, t=0$
$\therefore l=\int_1^0 \sqrt{t}\left(1-t^2\right)(-dt)$
$=\int_0^1\left(t^{1/2}-t^{5/2}\right) dt$
$=\left(\frac{t^{3/2}}{\frac{3}{2}}-\frac{t^{7/2}}{\frac{7}{2}}\right)$
$=\left(\frac{2}{3} t^{3/2}-\frac{2}{7} t^{7/2}\right)_0^1$
$=\left(\frac{2}{3}-\frac{2}{7}\right)-(0-0)$
$=\frac{14-6}{21}=\frac{8}{21}$

Question163

$\int\limits_0^{\frac{\pi}{4}} x . \sec^2 x dx=$ **MHT CET 2018**

Options:

A. $\frac{\pi}{4}+\log \sqrt{2}$

B. $\frac{\pi}{4}-\log \sqrt{2}$

C. $1+\log \sqrt{2}$

D. $1-\frac{1}{2} \log 2$

Answer: B

Solution:

Integrating by parts

$$\int_0^{\frac{\pi}{4}} x \sec^2 x\, dx=\left(x \int \sec^2 x dx\right)_0^{\frac{\pi}{4}}-\int_0^{\frac{\pi}{4}}\left(\frac{d}{dx}(x) . \int \sec^2 x\, dx\right) dx$$

$$=(x \tan x)_0^{\frac{\pi}{4}}-\int_0^{\frac{\pi}{4}} \tan x\, dx$$

$$\Rightarrow \mathrm{I}=(x \tan x-\ell n \sec x)_0^{\frac{\pi}{4}}=\frac{\pi}{4}(1)-\ell n \sqrt{2}$$

$$=\frac{\pi}{4}-\log \sqrt{2}$$

Question164

If $\int\limits_0^k \frac{dx}{2+18 x^2}=\frac{\pi}{24}$ **, then the value of** k **is MHT CET 2018**

Options:

A. 3

B. 4

C. $\frac{1}{3}$

D. $\frac{1}{4}$

Answer: C

Solution:

$$\frac{1}{2}\int_{0}^{k}\frac{dx}{1+9x^2}=\frac{\pi}{24}$$

$$\Rightarrow \int_{0}^{k}\frac{dx}{1+(3x)^2}=\frac{\pi}{12}$$

$$\Rightarrow \left(\frac{1}{3}\tan^{-1}(3x)\right)_0^k=\frac{\pi}{12}$$

$$\Rightarrow \tan^{-1}3k=\frac{\pi}{4}$$

$$\Rightarrow 3k=1$$

$$\Rightarrow k=\frac{1}{3}$$

Question165

$\int_0^1 x\tan^{-1}x dx=$ **MHT CET 2017**

Options:

A. $\frac{\pi}{4}+\frac{1}{2}$

B. $\frac{\pi}{4}-\frac{1}{2}$

C. $\frac{1}{2}-\frac{\pi}{4}$

D. $-\frac{\pi}{4}-\frac{1}{2}$

Answer: B

Solution:

$$\int_0^1 x\tan^{-1}x dx=\left[\tan^{-1}x\int x\,dx\right]_0^1-\int_0^1\left(\frac{d}{dx}\tan^{-1}x\int x\,dx\right)dx$$

(uv rule)

$$=\left(\tan^{-1}x\cdot\frac{x^2}{2}\right)_0^1-\int_0^1\frac{1}{1+x^2}\cdot\frac{x^2}{2}dx$$

$$=\left(\frac{\pi}{4}\cdot\frac{1}{2}-0\right)-\frac{1}{2}\int_0^1\frac{x^2}{1+x^2}dx$$

$$=\frac{\pi}{8}-\frac{1}{2}\int_0^1\left(1-\frac{1}{1+x^2}\right)dx$$

$$=\frac{\pi}{8}-\frac{1}{2}\left[x-\tan^{-1}x\right]_0^1$$

$$=\frac{\pi}{8}-\frac{1}{2}\left[(1-0)-\left(\frac{\pi}{4}-0\right)\right]=\frac{\pi}{8}-\frac{1}{2}+\frac{\pi}{8}=\frac{\pi}{4}-\frac{1}{2}$$

Question166

If $\int_0^{\frac{\pi}{2}} \log \cos x \, dx = \frac{\pi}{2} \log\left(\frac{1}{2}\right)$, then $\int_0^{\frac{\pi}{2}} \log \sec x \, dx =$ MHT CET 2017

Options:

A. $\frac{\pi}{2} \log\left(\frac{1}{2}\right)$

B. $1 - \frac{\pi}{2} \log\left(\frac{1}{2}\right)$

C. $1 + \frac{\pi}{2} \log\left(\frac{1}{2}\right)$

D. $\frac{\pi}{2} \log 2$

Answer: D

Solution:

$$\int_0^{\frac{\pi}{2}} \log \sec x \, dx = \int_0^{\frac{\pi}{2}} \log\left(\frac{1}{\cos x}\right) dx$$
$$= -\int_0^{\frac{\pi}{2}} \log(\cos x) \, dx$$
$$= -\frac{\pi}{2} \log\left(\frac{1}{2}\right) = \frac{\pi}{2} \log 2$$

Question167

$\int_0^3 [x] dx =$ ______, where $[x]$ is greatest integer function MHT CET 2017

Options:

A. 3

B. 0

C. 2

D. 1

Answer: A

Solution:

$$\int_0^3 [x] \, dx = \int_0^1 0 \, dx + \int_1^2 1 \, dx + \int_2^3 2 \, dx$$
$$= [x]_1^2 + 2[x]_2^3$$
$$=(2-1)+2(3-2)$$
$$= 1 + 2$$
$$= 3$$

Question168

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \log\left(\frac{2-\sin\ x}{2+\sin\ x}\right)dx =$ **MHT CET 2016**

Options:

A. 1

B. 3

C. 2

D. 0

Answer: D

Solution:

Let $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \log\ \left(\frac{2-\sin\ x}{2+\sin\ x}\right)dx$

As given function $f(x) = \log\left(\frac{2-\sin x}{2+\sin x}\right)$ is odd.

$\Rightarrow I = \int f(x)\, dx = 0$

Question169

$\int_{0}^{\frac{\pi}{2}} \left(\frac{\sqrt[n]{\sec\ x}}{\sqrt[n]{\sec\ x}+\sqrt[n]{\operatorname{cosec}\ x}}\right)dx =$ **MHT CET 2016**

Options:

A. $\frac{\pi}{2}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{6}$

Answer: C

Solution:

$\int_{0}^{\frac{\pi}{2}} \left(\frac{\sqrt[n]{\sec\ x}}{\sqrt[n]{\sec\ x}+\sqrt[n]{\cos\ ec\ x}}\right)dx$ ..... (i)

$\Rightarrow\ \ I = \int_{0}^{\frac{\pi}{2}} \frac{\sqrt[n]{\sec\ \left(\frac{\pi}{2}-x\right)}}{\sqrt[n]{\sec\ \left(\frac{\pi}{2}-x\right)+\sqrt[n]{\operatorname{cosce}\left(\frac{\pi}{2}-x\right)}}}dx$

$\Rightarrow\ \ I = \int_{0}^{\frac{\pi}{2}} \frac{\sqrt[n]{\operatorname{cosce}\ x}}{\sqrt[n]{\operatorname{cosce} x}+\sqrt[n]{\sec\ x}}dx$ ..... (ii)

Adding equation (i) and (ii)

$$2I = \int_0^{\frac{\pi}{2}} dx$$

$$\Rightarrow \quad 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow \quad \frac{\pi}{4}$$

Question170

The value of $\int_0^\pi \log(1 + \cos x) dx$ is MHT CET 2012

Options:

A. $-\frac{\pi}{2}\log 2$

B. $\pi \log \frac{1}{2}$

C. $\pi \log 2$

D. $\frac{\pi}{2}\log 2$

Answer: B

Solution:

$$I = \int_0^\pi \log(1 + \cos x) dx \ldots (i)$$

Let $I = \int_0^\pi \log\{1 + \cos(\pi - x)\} dx$

$$= \int_0^\pi \log(1 - \cos x) dx \ldots (ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^\pi \{\log(1 + \cos x) + \log(1 - \cos x)\} dx$$

$$I = \frac{1}{2}\int_0^\pi \log(1 - \cos^2 x) dx$$

$$= \frac{1}{2}\int_0^\pi \log \sin^2 x dx$$

$$= \int_0^\pi \log \sin x dx$$

$$= 2\int_0^{\pi/2} \log \sin x dx$$

$$\left\{\because \int_0^{2a} f(x) dx = 2\int_0^a f(x) dx, \text{ if } f(2a - x) = f(x)\right\}$$

$$= 2\left\{-\frac{\pi}{2}\log 2\right\}$$

$$\left(\because \int_0^{\pi/2} \log \sin x dx = -\frac{\pi}{2}\log 2\right)$$

$$= \pi \log \frac{1}{2}$$

Question171

The value of $\int_3^4 \sqrt{(4 - x)(x - 3)} dx$ is MHT CET 2012

Options:

A. $\frac{\pi}{16}$

B. $\frac{\pi}{8}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{2}$

Answer: B

Solution:

Let $I=\int_3^4 \sqrt{(4-x)(x-3)} d x$

$=\int_3^4 \sqrt{-x^2+7 x-12} d x$

$=\int_3^4 \sqrt{-\left(x^2-7 x+\frac{49}{4}-\frac{49}{4}\right)-12} d x$

$=\int_3^4 \sqrt{-\left(x-\frac{7}{2}\right)^2+\frac{49}{4}-12} d x$

$=\int_3^4 \sqrt{\frac{1}{4}-\left(x-\frac{7}{2}\right)^2} d x$

Let $t=x-\frac{7}{2} \Rightarrow d t=d x$

$\therefore$ Upper limit $=\frac{1}{2}$ and lower limit $=-\frac{1}{2}$

$$
\begin{aligned}
& =\int_{-1 / 2}^{1 / 2} \sqrt{\left(\frac{1}{2}\right)^2-t^2} d t \\
& =2 \int_0^{1 / 2} \sqrt{\left(\frac{1}{2}\right)^2-t^2} d t \\
& =2\left[\frac{t}{2} \sqrt{\frac{1}{4}-t^2}+\frac{1}{8} \sin ^{-1} 2 t\right]_0^{1 / 2} \\
& =2\left[0+\frac{1}{8} \times \frac{\pi}{2}\right] \\
& =\frac{\pi}{8}
\end{aligned}
$$

Question172

The value of $\int_0^1 x^2\left(1-x^2\right)^{3 / 2} d x$ is MHT CET 2012

Options:

A. $\frac{1}{32}$

B. $\frac{\pi}{8}$

C. $\frac{\pi}{16}$

D. $\frac{\pi}{32}$

Answer: D

Solution:

Let $I=\int_0^1 x^2\left(1-x^2\right)^{3/2} dx$

$$\left(\text{let } x=\sin\theta \Rightarrow dx=\cos\theta d\theta\right)$$

$$I=\int_0^{\pi/2} \sin^2\theta\left(\cos^2\theta\right)^{3/2}\cdot\cos\theta d\theta$$

$$=\int_0^{\pi/2} \sin^2\theta\cdot\cos^4\theta d\theta$$

By Gamma function, $I=\frac{\sqrt{\frac{2+1}{2}}\left|\frac{4+1}{2}\right.}{2\left|\frac{2+4+2}{2}\right.}=\frac{\sqrt{\frac{3}{2}}\sqrt{\frac{5}{2}}}{2\Gamma 4}$

$=\frac{\frac{1}{2}\left|\frac{1}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\cdot\sqrt{\frac{1}{2}}\right.}{2\cdot 3\cdot 2}$

$=\frac{3\cdot\sqrt{\pi}\cdot\sqrt{\pi}}{2\cdot 2\cdot 2\cdot 2\cdot 3\cdot 2}=\frac{\pi}{32}$

Question173

$\int_{-\pi/2}^{-\pi/2} \frac{\cos x}{1+e^x} dx$ is equal to MHT CET 2011

Options:

A. 1

B. 0

C. -1

D. None of these

Answer: A

Solution:

$I=\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x} dx$

$$I=\int_{-\pi/2}^{\pi/2} \frac{\cos(\pi/2-\pi/2-x)}{1+e^{(\pi/2-\pi/2-x)}} dx$$

$$=\int_{-\pi/2}^{\pi/2} \frac{\cos(-x)}{1+e^{-x}} dx$$

$$I=\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^{-x}} dx$$

$$=\int_{-\pi/2}^{\pi/2} \frac{e^x\cos x}{1+e^x} dx$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_{-\pi/2}^{\pi/2} \frac{(1+e^x)\cos x}{(1+e^x)} dx$$
$$= \int_{-\pi/2}^{\pi/2} \cos x dx$$
$$= 2\int_0^{\pi/2} \cos x dx$$

[Since, $\cos x$ is an even function.]

$$\therefore \quad 2I = 2[\sin x]_0^{\pi/2} = 2(1-0) = 2$$
$$\Rightarrow \quad I = 1$$

Question174

$\int_0^{\pi/2} \frac{dx}{1+\tan x}$ **is equal to MHT CET 2011**

Options:

A. π

B. $\pi/2$

C. $\pi/3$

D. $\pi/4$

Answer: D

Solution:

Given, $I = \int_0^{\pi/2} \frac{dx}{1+\tan x}$

$$\begin{aligned} I &= \int_0^{\pi/2} \frac{\cos x}{\sin x+\cos x} dx \quad \ldots \\ I &= \int_0^{\pi/2} \frac{\cos(\pi/2-x)}{\sin(\pi/2-x)+\cos(\pi/2-x)} dx \\ &= \int_0^{\pi/2} \frac{\sin x}{\cos x+\sin x} dx \quad \ldots \end{aligned}$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\pi/2} \left(\frac{\sin x+\cos x}{\sin x+\cos x}\right) dx$$
$$= \int_0^{\pi/2} dx = \pi/2$$
$$\Rightarrow \quad I = \pi/4$$

Question175

By Simpson rule taking $n=4$, the value of the integral $\int_0^1 \frac{1}{1+x^2} dx$ is equal to MHT CET 2011

Options:

A. 0.788

B. 0.781

C. 0.785

D. None of these

Answer: C

Solution:

Here, $h=1/4,=0.25, y=\frac{1}{1+x^2}$

	x	y
1	0	1.0
2	0.25	0.941
3	0.5	0.8
4	0.75	0.64
5	1	0.5

	x	y
1	0	1.0
2	0.25	0.941
3	0.5	0.8
4	0.75	0.64
5	1	0.5

By Simpson's Rule

$$\int_0^1 \frac{dx}{1+x^2}=\frac{1}{4\times 3}$$
$$[(1+0.5)+4(0.941+0.64)+2(0.8)]$$
$$=\frac{1}{12}[9.424]=0.785$$

Question176

If $\int_0^1 \tan^{-1} x dx=p$, then the value of $\int_0^1 \tan^{-1}\left(\frac{1-x}{1+x}\right) dx$ is MHT CET 2010

Options:

A. $\frac{\pi}{4}+p$

B. $\frac{\pi}{4}-p$

C. $1+p$

D. $1-p$

Answer: B

Solution:

$$\int_0^1 \left(\frac{1-x}{1+x}\right) dx$$

$$= \int_0^1 \left[\tan^{-1}(1) - \tan^{-1}(x)\right] dx$$

$$= \int_0^1 \frac{\pi}{4} dx - \int_0^1 \tan^{-1}(x) dx$$

$$= \frac{\pi}{4} - p$$

Question177

The value of $\int_0^{\pi/2} \log(\operatorname{cosec} x) dx$ is MHT CET 2010

Options:

A. $\frac{\pi}{2}\log 2$

B. $\pi \log 2$

C. $-\frac{\pi}{2}\log 2$

D. $2\pi \log 2$

Answer: A

Solution:

Let $I = \int_0^{\pi/2} \log(\operatorname{cosec} x) dx$

$= \int_0^{\pi/2} \log\left(\frac{1}{\sin x}\right) dx$

$= -\int_0^{\pi/2} \log \sin x dx$

$= \frac{\pi}{2}\log 2$

$\left[\because \int_0^{\pi/2} \log \sin x dx = -\frac{\pi}{2}\log 2\right]$

Question178

The value of $\int_4^7 \frac{(11-x)^2}{x^2+(11-x)^2} dx$ is MHT CET 2010

Options:

A. 1

B. 1/2

C. 3/2

D. 0

Answer: C

Solution:

Let

$$I = \int_4^7 \frac{(11-x)^2}{x^2+(11-x)^2} dx$$
$$= \int_4^7 \frac{x^2}{(11-x)^2+x^2} dx$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_4^7 1 dx = [x]_4^7 = 3$$

$$\Rightarrow \quad I = \frac{3}{2}$$

Question179

Which of the following is true? MHT CET 2009

Options:

A. $\int_0^1 e^x dx = e$

B. $\int_0^1 2^x dx = \log 2$

C. $\int_0^1 \sqrt{x} dx = \frac{2}{3}$

D. $\int_0^1 x dx = \frac{1}{3}$

Answer: C

Solution:

$\int_0^1 e^x dx = [e^x]_0^1 = e - 1$

(b) $\int_0^1 2^x dx = \left[\frac{2^x}{\log_e 2}\right]_0^1 = \frac{1}{\log 2} \cdot (2 - 2^0) = \frac{1}{\log 2}$

(c) $\int_0^1 \sqrt{x} dx = \left[\frac{x^{3/2}}{3/2}\right]_0^1 = \frac{2}{3}$

(d) $\int_0^1 x dx = \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2}$

Question180

$\int_0^{\pi/2} \frac{\sin x-\cos x}{1-\sin x \cdot \cos x} dx$ **is equal to MHT CET 2009**

Options:

A. 0

B. $\frac{\pi}{2}$

C. $\frac{\pi}{4}$

D. π

Answer: A

Solution:

Let $I=\int_0^{\pi/2} \frac{\sin x-\cos x}{1-\sin x \cos x} dx$

On putting $x=\left(\frac{\pi}{2}-x\right)$ in Eq. (i), we get

$$\begin{aligned} I &= \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2}-x\right)-\cos\left(\frac{\pi}{2}-x\right)}{1-\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)} dx \\ &= \int_0^{\pi/2} \frac{\cos x-\sin x}{1-\sin x \cos x} dx \\ &= -\int_0^{\pi/2}\left(\frac{\sin x-\cos x}{1-\sin x \cos x}\right) dx \end{aligned}$$

On adding Eqs. (i) and (ii), we get

$$2I=\int_0^{\pi/2} 0 dx=0$$

$\Rightarrow \quad I=0$

Question181

The value of $\int_{\pi/4}^{\pi/2} e^x(\log \sin x+\cot x) dx$ **is MHT CET 2008**

Options:

A. $e^{\pi/4} \log 2$

B. $-e^{\pi/4} \log 2$

C. $\frac{1}{2} e^{\pi/4} \log 2$

D. $-\frac{1}{2} e^{\pi/4} \log 2$

Answer: C

Solution:

$$I = \int_{\pi/4}^{\pi/2} e^x(\log \sin x \quad + \cot x)dx$$

$$\Rightarrow I = \int_{\pi/4}^{\pi/2} e^x \log \sin x \quad dx$$

$$+ \int_{\pi/4}^{\pi/2} e^x \cot x dx$$

Let $= \int_{\pi/4}^{\pi/2} e^x \log \sin x dx + [e^x \log \sin x]_{\pi/4}^{\pi/2}$

$$- \int_{\pi/4}^{\pi/2} e^x \log \sin x dx$$

$$= e^{\pi/2} \log \sin \frac{\pi}{2} - e^{\pi/4} \log \sin \frac{\pi}{4}$$

$$= -e^{\pi/4} \log\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{1}{2} e^{\pi/4} \log 2$$

Question182

Considering four sub-intervals, the value of $\int_0^1 \frac{1}{1+x} dx$ by Trapezoidal rule, is MHT CET 2008

Options:

A. 0.6870

B. 0.6677

C. 0.6977

D. 0.5970

Answer: C

Solution:

i	x_i	$y_i = \frac{1}{1+x_i}$
0	0	1
1	0.25	0.8
2	0.5	0.67
3	0.75	0.571
4	1	0.5

Trapezoidal rule gives

$$\int_0^1 \frac{1}{1+x} dx = \frac{h}{2}[y_0 + 2(y_1 + y_2 + y_3) + y_4]$$

$$= \frac{1-0}{2 \times 4}[1 + 2(0.8 + 0.67$$

$$= 0.6977$$

Question183

By Simpson's rule, the value of $\int_1^2 \frac{dx}{x}$ dividing the interval (1, 2) into four equal parts, is MHT CET 2008

Options:

A. 0.6932

B. 0.6753

C. 0.6692

D. 7.1324

Answer: A

Solution:

$$h = \frac{2-1}{4} = \frac{1}{4}$$

$$\text{Now, } x_0 = 1, x_1 = 1 + \frac{1}{4}, x_2 = 1 + 2 \times \frac{1}{4},$$
$$x_3 = 1 + 3 \times \frac{1}{4}, x_4 = 1 + 4 \times \frac{1}{4}$$

$$\begin{aligned} \text{ie,} \quad & x_0 = 1, x_1 = 1.25, x_2 = 1.5, x_3 \\ & = 1.75, x_4 = 2 \\ \Rightarrow \quad & y_0 = 1, y_1 = 0.8, y_2 = 0.667, y_3 \\ & = 0.571, y_4 = 0.5 \end{aligned}$$

$\therefore$ Using Simpson's $\frac{1}{3}$ rd rule

$$\begin{aligned} \int_1^2 \frac{dx}{x} &= \frac{1}{12}[(1+0.5) + 4(0.8+0.571) \\ &\quad + 2(0.667)] \\ &= \frac{1}{12}[1.5 + 5.484 + 1.334] \\ &= \frac{1}{12}[8.318] = 0.6932 \end{aligned}$$

Question184

The value of $\int_0^\pi x \sin^3 x dx$ is MHT CET 2008

Options:

A. $\frac{4\pi}{3}$

B. $\frac{2\pi}{3}$

C. 0

D. None of these

Answer: B

Solution:

Let $I=\int_0^\pi x\sin^3 x dx$ ...(i)

Also, $I=\int_0^\pi(\pi-x)\sin^3 x dx$ ...(ii)

On adding Eqs. (i) and (ii), we get

$$\begin{aligned} 2I &= \pi\int_0^\pi \sin^3 x dx \\ &= \frac{\pi}{4}\int_0^\pi (3\sin x-\sin 3x)dx \\ &= \frac{\pi}{4}\left[-3\cos x+\frac{\cos 3x}{3}\right]_0^\pi \\ &= \frac{\pi}{4}\left[3-\frac{1}{3}+3-\frac{1}{3}\right]=\frac{4\pi}{3} \\ \text{Hence, } I &= \frac{2\pi}{3} \end{aligned}$$

Question185

The value of $\int_0^{\pi/2}\frac{\cos 3x+1}{2\cos x-1}dx$ is MHT CET 2007

Options:

A. 2

B. 1

C. $\frac{1}{2}$

D. 0

Answer: B

Solution:

Let

$$I=\int_0^{\pi/2}\frac{\cos 3x+1}{2\cos x-1}dx$$

$$=\int_0^{\pi/2}\frac{\cos 3x-\cos\frac{3\pi}{3}}{2\left(\cos x-\cos\frac{\pi}{3}\right)}dx$$

$$=\int_0^{\pi/2}\frac{\left(4\cos^3 x-3\cos x\right)-\left(4\cos^3\frac{\pi}{3}-3\cos\frac{\pi}{3}\right)}{2\left(\cos x-\cos\frac{\pi}{3}\right)}dx$$

$$=2\int_0^{\pi/2}\left(\frac{\cos^3 x-\cos^3\frac{\pi}{3}}{\cos x-\cos\frac{\pi}{3}}\right)dx$$

$$=\frac{3}{2}\int_0^{\pi/2}\left(\frac{\cos x-\cos\frac{\pi}{3}}{\cos x-\cos\frac{\pi}{3}}\right)dx$$

$$=\int_0^{\pi/2}\left(1+\cos 2x+\frac{1}{2}+\cos x\right)dx-\frac{3\pi}{4}$$

$$=\frac{3\pi}{4}+1-\frac{3\pi}{4}=1$$

Question186

The value of $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx$ is MHT CET 2007

Options:

A. 1

B. 0

C. -1

D. None of these

Answer: B

Solution:

$$I = \int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx$$
$$= \int_0^1 \tan^{-1}\left(\frac{x+x-1}{1-x(x-1)}\right) dx$$
Let $$= \int_0^1 \tan^{-1} x dx + \int_0^1 \tan^{-1}(x-1) dx$$
$$= \int_0^1 \tan^{-1} x dx + \int_0^1 \tan^{-1}(1-x-1) dx$$
$$= \int_0^1 \tan^{-1} x dx - \int_0^1 \tan^{-1} x dx = 0$$

Question187

By the application of Simpson's one-third rule for numerical integration, with two subintervals, the value of $\int_0^1 \frac{dx}{1+x}$ is MHT CET 2007

Options:

A. $\frac{17}{36}$

B. $\frac{17}{25}$

C. $\frac{25}{36}$

D. $\frac{17}{24}$

Answer: C

Solution:

Since, the given integration is divided into two subintervals.ie,

$$h = \frac{1-0}{2} = \frac{1}{2}$$

$$\therefore \quad \int_0^1 \frac{1}{1+x} dx = \frac{h}{3}\left[\left(y_0+y_2\right)+4\left(y_1\right)\right]$$

At

$$x = 0, y = 1$$
$$x = \frac{1}{2}, y_1 = \frac{2}{3}$$

and

$$x = 1, y_2 = \frac{1}{2}$$

$$\therefore \int_0^1 \frac{1}{1+x} dx = \frac{1}{2 \cdot 3}\left[\left(1 + \frac{1}{2}\right) + 4\left(\frac{2}{3}\right)\right]$$
$$= \frac{1}{6}\left[\frac{3}{2} + \frac{8}{3}\right] = \frac{25}{36}$$
